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Choice of sample size for high transport critical current density in a granular superconductor: percolation versus self-field effects

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Abstract. The percolative character of the current paths and the self-field effects were considered to estimate optimal sample dimensions for the transport current of a granular superconductor by means of a Monte Carlo algorithm and critical-state model calculations. We showed that, under certain conditions, self-field effects are negligible and the J_c dependence on sample dimensions is determined by the percolative character of the current. Optimal dimensions are demonstrated to be a function of the fraction of superconducting phase in the sample.

1. Introduction

Since the discovery of high-temperature superconductors (HTCs) by Bednorz and Müller [1] the transport critical current density J_c (i.e. the maximum current density a superconductor can carry without dissipation) has been one of the most studied parameters of these materials, especially because of its importance when considering commercial applications. In fact, it is believed that the low J_c of HTC materials is the basic problem to be solved in order to achieve industrial applications [2, 3].

Depending on the sample type, different limiting factors for J_c have been established. For example, it is well known [4–6] that ceramic superconductors consist of a collection of randomly oriented grains connected by weak links which are responsible for the low value of J_c in these materials. A different situation seems to arise in epitaxial thin films where 'micro' weak links appear within the grains as a result of other microstructural defects and limit J_c [7]. In other types of samples such as Bi–Sr–Ca–Cu–O tapes, different models have been proposed for the description of the field and temperature behaviour of J_c which take into account the relation between microstructure and the percolative path of the current [8–10].

The influence of sample geometry on J_c has also been studied, particularly in connection with self-field effects [11, 12]: since J_c strongly depends on the field, the self-field depresses it when the sample size increases.

In this work two effects were considered to estimate the dependence of the critical current density on sample geometry for a granular superconductor: the self-field effects mentioned above and the increase of J_c connected with the fact that, when the sample cross-section increases, a greater number of percolative paths appear in the superconductor. The latter effect, which is a consequence of percolation theory [13, 14] (usually applied to study the transport properties of ceramic superconductors [15, 16]), has been never explicitly considered, as far as we know, to describe the J_c dependence on sample size.

2. Theory

2.1. Self-field effects

It is accepted that the critical current density in granular superconductors strongly depends on the magnetic field [17]. Thus, their magnetic behaviour in the mixed state is usually described using the exponential model [18] or Kim's model [19] rather than the conventional Bean critical-state model [20]. In this section we shall review the general treatment developed in [11] to derive relationships between the critical current density and the sample dimensions and discuss some important consequences useful to understand the results presented in the next section.

The self-field generated at the edges of an infinite slab of thickness a with a transport current J flowing along its major axis can be written as

$$H_s = J\frac{a}{2} \tag{1}$$

and, defining (following [11]) J_c as the transport current density needed for full penetration of the sample by the self-field, we just have to calculate the field of full penetration

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from the critical-state model assumptions and Ampère's law. Mathews and Müller [11] found the following relationships between J_c and the sample thickness: for the exponential model

$$J_c = \frac{2H_0}{a} \ln\left(1 + \frac{J_{c0}a}{2H_0}\right) \tag{2}$$

and for Kim's model

$$J_c = \frac{2H_0}{a}\left[\left(\frac{J_{c0}a}{2H_0} + 1\right)^{1/2} - 1\right] \tag{3}$$

where J_{c0} is the critical density at zero field and H_0 is a parameter which represents the field needed to decrease J_c by a factor of e in the exponential model or by a factor of 2 in Kim's model.

It is important to note that both formulae have the same asymptotic behaviour for small thicknesses (i.e. J_c does not depend on sample size) while, for large thicknesses, J_c decreases following potential laws. It should be also pointed out that, in deriving formulae (2) and (3), the percolative character of the current path was not considered. If this character is taken into account, we propose that the critical current density at zero field can be expressed as

$$J_{c0} = J'_{c0} f(a, p) \tag{4}$$

where p is the fraction of superconducting phase present in the sample, J'_{c0} is the critical current density of an ideal sample in zero applied field (i.e. a sample with no defects or percolative current paths) and $f(a, p)$ is a dimensionless function which evaluates the effects of the percolative paths on the critical current density. To estimate this function we performed the simulation described below.

2.2. Simulation

We modelled the sample as a 'simple cubic' array of grains, each one of dimensions $L_0 \times L_0 \times a_0$ with probability p to be superconducting. This can be understood in three different ways depending on the feature limiting J_c: a fraction $1 - p$ of the grains have lower critical current than the fraction p remaining, because of intragranular 'micro' weak lines (a situation suitable for thin films), a fraction $1 - p$ of the grains are connected only by weak links with all their neighbours (a better description for ceramic superconductors) or a fraction $1 - p$ of the grains has lower critical temperature than the others (which is an appropriate description for both systems).

However, as our array is ideally regular, a quantitative extrapolation to ceramic samples must be done with caution, even if we consider that our results will remain qualitatively the same. A better description for real ceramics should be obtained by modelling their structure through the Swiss-cheese model [13, 14].

The percolative paths were found using a standard algorithm [21] in samples with different dimensions $L \times L \times a$. The variation of a and L allowed us to examine the range from two-dimensional (2D) samples with only one row of grains ($a = a_0$, $L = L_0$) to three-dimensional (3D)

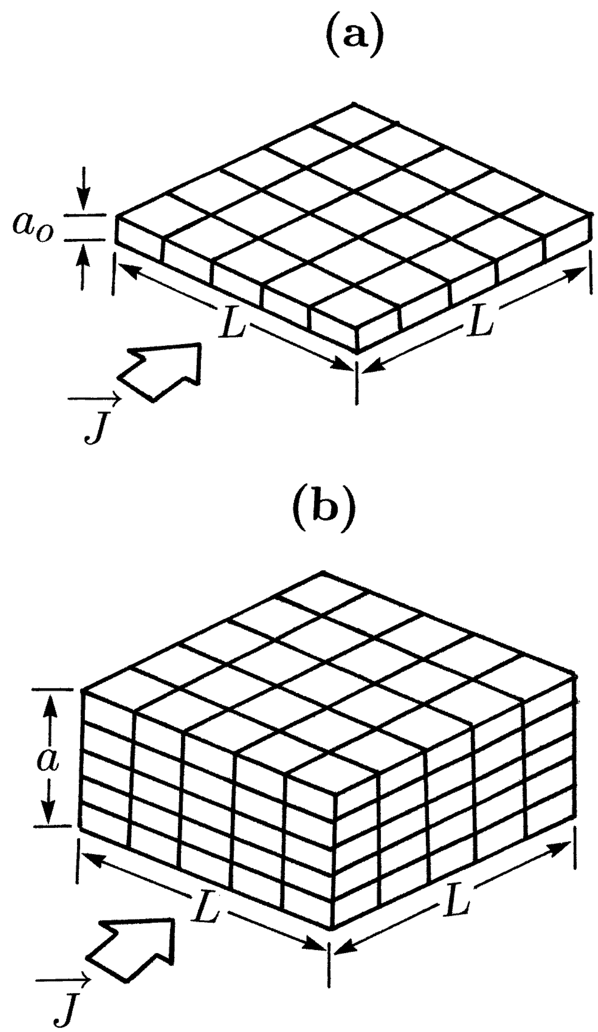

Figure 1. Sketch of typical arrays used in the simulation: (*a*) $L = 5L_0$, $a = a_0$; (*b*) $L = 5L_0$, $a = 5a_0$.

samples ($a = Na_0$, $L = NL_0$) where N is the number of grains in a given direction (see figure 1). J_{c0} was estimated as in [22] by finding the plane perpendicular to the flow of current where a minimum number of percolative paths was found (limiting plane) and defining the critical current of the sample as proportional to the number of paths piercing the limiting plane, assuming that each one supports a current density J'_{c0} without dissipation. This implicitly means that all grains or links belonging to the percolative paths have the same critical current density. A similar procedure has been followed by Octavio *et al* [23] although, in their case, the critical current was estimated through the minimum neck of the current paths in the Swiss-cheese model. However, it is worth noticing that our procedure is valid when the transport of current is isotropic, i.e. if the current does not have a preferential direction of flow.

Finally, a comment on the sources of errors in our simulation and how we handled them is necessary. Although impossible to eliminate, finite-size effect were delimited by studying samples with different sizes. Fluctuation effects due to different possible configurations for each value of p and because of dead ends were also

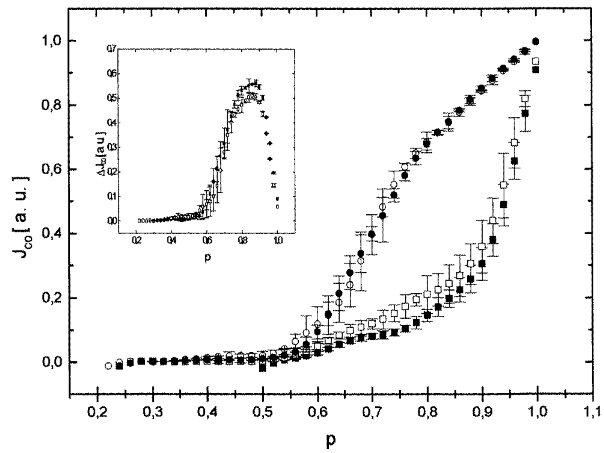

Figure 2. J_{c0} versus p dependences for 2D (□, ■) and 3D (○, ●) samples, for N = 20 (□, ○) and N = 30 (■, ●). The insert shows the difference between the upper and the lower curves.

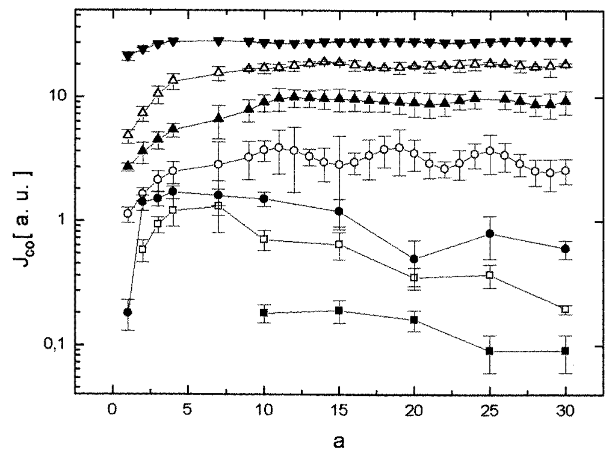

Figure 3. J_{c0} versus a dependences for different fractions of the superconducting phase, p. From the upper curve to the bottom, p = 0.96, 0.88, 0.76, 0.60, 0.54, 0.48 and 0.36.

diminished by repeating each calculation several times and averaging. At the end we obtained the following approximate relative error, e_r, for different intervals of p:

$$0.3 \leq p < 0.5 \qquad e_r \approx 10\%$$

$$0.5 \leq p < 0.7 \qquad e_r \approx 20\%$$

$$0.7 \leq p < 0.9 \qquad e_r \approx 15\%$$

$$0.9 \leq p \leq 1 \qquad e_r \approx 5\%.$$

3. Results and discussion

Figure 2 shows the dependences of J_{c0} on the fraction of superconducting phase, p, for different sample dimensions. Note the different behaviours of 2D samples and 3D samples that, as already mentioned, is a consequence of the better connectivity in 3D samples because of the greater number of neighbours each grain has. As expected from percolation theory, J_{c0} is zero for $p < 0.3$ in the case of 3D samples, and for $p < 0.6$ in the case of 2D samples. However, an abrupt change of slope of J_{c0} close to $p = 0.6$ was found even in 3D samples, perhaps as a result of the intrinsic order of our array and the 2D character our definition of J_c has. So, this result should be extrapolated with caution to real ceramics where the grains are randomly oriented as previously pointed out. In the insert of figure 2 the difference between J_{c0} in 3D samples and 2D samples is displayed. It clearly exhibits a maximum around $p = 0.8$ that strongly influences some of the results presented below.

Figure 3 shows J_{c0} versus a for different values of p for samples with $N = 30$ (similar results were obtained for $N = 20$ and $N = 15$). For $p > 0.6$ a saturation in J_{c0} appears, indicating that the critical current increases like the sample cross-section. For $p < 0.6$, the curves have well-defined maxima which can be explained if we assume that about some a the number of conducting paths in the limiting plane increases more slowly than the sample cross-section. The behaviour of the curve for $p = 0.36$ is not as clear since no percolation is achieved for $a < 10$, suggesting the existence of some threshold dimension above which, for a given p, percolative conduction takes place. A similar threshold dimension was also reported by Octavio *et al* [24] in a different context.

The self-field effects were calculated from equations (2) and (3) by substituting the results from our J_{c0} simulations. Two limiting cases should be considered. Firstly,

$$\frac{J_{c0}a}{2H_0} \gg 1 \tag{5}$$

which implies that $J_c \sim 1/a^{\alpha}$ ($\alpha = 1$ for the exponential model and $\alpha = 1/2$ for Kim's model). Then, for lower values of the film thickness, greater values of J_c will be achieved, as is commonly accepted from self-field analysis. The second case is

$$\frac{J_{c0}a}{2H_0} \ll 1 \tag{6}$$

which implies that

$$J_c = \beta J_{c0} \tag{7}$$

where $\beta = 1$ for the exponential model and $\beta = 1/2$ for Kim's model. Then, optimal dimensions would be determined by $f(a, p)$.

Figure 4 shows the a_s/L versus p plots for $N = 20$ and $N = 30$, where a_s is the sample thickness for which J_{c0} is a maximum or saturates as plotted in figure 3 and N is the number of grains in the direction of the current flow. The errors ($\pm a_0/L$) are not plotted in the figure for clarity. It is shown that for high-quality samples ($p > 0.85$) very thin samples are ideal for obtaining high J_{c0} values. For intermediate-quality samples ($0.6 < p < 0.85$) where greater difference appear between J_{c0} in 3D samples and 2D samples the better values of J_{c0} are achieved for $a_s \approx 0.5L$. For very poor samples ($p < 0.4$) the highest J_{c0} appear for very bulk samples where more percolative paths could appear. If the condition (7) is fulfilled the previous results for J_{c0} are the same for J_c, and thus they suggest simple rules for obtaining high values of J_c in granular superconductors.

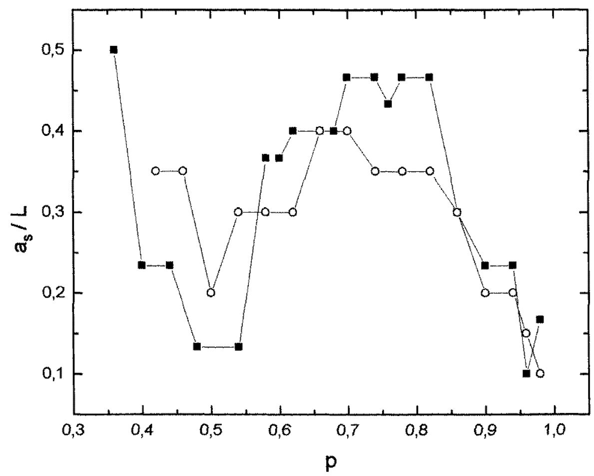

Figure 4. a/L versus dependences for N = 20 (○) and N = 30 (■).

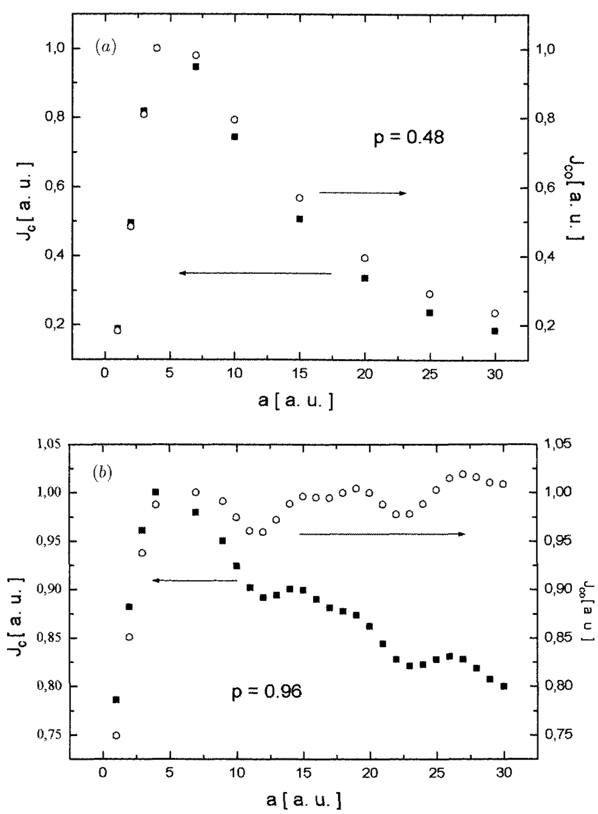

Figure 5. J_c versus a and J_{c0} versus a dependences for different values of p: (a) p = 0.48; (b) p = 0.96.

The influence of the self-field effects (i.e. the comparison between J_c and J_{c0}) is plotted in figure 5 for two characteristic values of p, using $H_0 = 100$ A cm^{-1}, $J_{c0} = 500$ A cm^{-2} and $a_0 = 10^{-3}$ cm. From them, we can argue that self-field effects are more visible for high values of p, for which J_c saturates instead of decreasing.

Finally, figure 6 shows the difference between J_{c0} and J_c as a function of $J'_{c0}/2H_0$ for different values of $f(a, p)$. This makes it evident that the influence of the self-field is

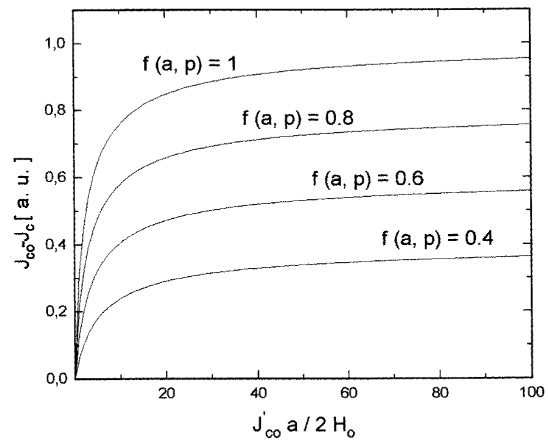

Figure 6. $J_{c0} - J_c$ versus $J'_{c0}a/2H_0$ for different values of $f(a, p)$.

determined by $J'_{c0}a/2H_0$ for the range entire of percolation possibilities: the lower it is, the lower will be the self-field effects.

4. Conclusions

We showed the influence of the percolative current paths in the J_c dependence on sample dimensions of granular superconductors. We demonstrated that, under certain conditions, self-field effects dominate the J_c dependence on sample dimensions. However, if these conditions are not fulfilled, J_c is determined by the percolative character of the current. This means that, even when self-field effects are disregarded, the critical current density is thickness dependent. We proposed optimal sample dimensions as a function of the fraction of the superconducting phase in the sample using realistic parameters. Two extremes are well defined: samples with high superconducting fractions show the highest critical current densities when very thin, while bulk samples are demonstrated to perform better if the superconducting fraction is low.

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