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Avalanche behavior in one-dimensional superconductors with a periodic distribution of pinning centers: a Monte Carlo approach

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Abstract

Through Monte Carlo simulations, and using a suitable probability, we studied the dynamic formation of the Bean's critical state as the applied field increases in a one-dimensional superconductor with a periodic distribution of pinning centers. We considered an applied field acting on one of the surfaces of the sample and we counted the number of vortices exiting out from the other surface. We found a threshold response time, separating the region where criticality was present from the region where it was not. Defining an avalanche as the number of vortices who left the superconductor from one critical state to the next one, we also noted the absence of characteristic sizes and times of avalanches, in a wide range of response times, allowing us to analyze our results in the framework of the self-organized criticality theory. Some contradictions between our simulations and that theory are explained based on the simplicity of our model. © 1997 Elsevier Science B.V.

1. Introduction

The similarity between the critical state models in type-II superconductors and sandpiles was first pointed out by de Gennes [1] in 1966. More than 20 years later Bak et al. [2,3] developed what is now known as the self-organized criticality theory (SOC) which tries to explain the behavior of some complex systems as sandpiles, earthquakes, etc. These systems show a $s^{-\alpha}$ distribution of avalanche sizes, a $t^{-\nu}$ distribution of duration of avalanches and a $f^{-\beta}$ power spectrum.

The early work of Vinokur et al. [4] is specially relevant in the field of superconductivity, since they showed that, if the pinning potential depends logarithmically on the current density, the system exhibits SOC behavior. In 1993, Wang and Shi [5,6] reported experimental evidence for flux avalanches in high-T_c superconductors from relaxation experiments in BiSrCaCuO and YBaCuO single crystals. In 1994, Field et al. [7] designed an experiment to check if the behavior of type-II superconductors could be described by the SOC theory, as seems to happen for sandpiles [8]. They submitted a niobium hollow cylinder to an increasing axial magnetic field and measured the flux penetration in the cylinder's hole. Their results showed a good coincidence with the SOC predictions. More recently Nowak et al. [9] designed a similar experiment to study the flux dynamics of Nb rings at different temperatures. They found a crossover from a broad distribution of avalanche sizes to a narrow distribution of system spanning events.

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A rich variety of computer simulations have been carried out to elucidate which models exhibit SOC behavior (see, for example, the work of Kadanoff et al. [10]) and to find out which phenomena could be described by the SOC theory [11–13]. In the field of superconductivity, Pla et al. [13] have found avalanche behavior in the vortex lattice of type-II superconductors close to the pinning–depinning transition through molecular dynamic techniques.

In this work we report and discuss Monte Carlo simulations for a one-dimensional superconductor with a periodic distribution of pinning centers in a system arranged in order to mimic the experiment described in Ref. [6] assuming a "vortex-glass-like" [14] pinning potential.

2. The simulation

We considered a one-dimensional superconductor with 32 pinning centers in the presence of an applied field acting on one of its surfaces. The external field was increased by ΔH every t_s Monte Carlo step (m.c.s.) as defined below. Immediately, a proportional number of vortices was introduced at the corresponding sample surface in order to satisfy the boundary condition. We chose the vortex-glass-like probability:

$$P_{i\to i+1} = \begin{cases} e^{-\eta\frac{j_c}{j}} & j > j_c \\ 0 & j < j_c \end{cases} \quad (1)$$

as the vortex probability to jump from the pinning center i to the $i+1$, where j is the local current density, which is proportional to the number of vortices in the $(i+1)$th pinning center minus the number of vortices in the ith pinning center [15,16], j_c is the critical current density of the superconductor, and $\eta = U_0/kT$ is a simulation parameter related to the temperature and to the depth of the pinning centers of our system.

Our algorithm can be described as follows.

(1) We fixed a field in one of the superconductor boundaries and we calculated the probability, Eq. (1), for all the pinning centers. *This operation defines our m.c.s.*

(2) Every time the probability was calculated, it was compared with a random number. If the probability was greater than the random number, one vortex was removed from the ith pinning center and added to the $(i+1)$th.

(3) We compared j and j_c for all the pinning centers. If $j = j_c$ for all of them, we said the system was in the critical state, and we counted the number of vortices that left the superconductor until a new critical state was reached. We called this number the *avalanche size*, and the number of iterations needed to reach one critical state from another, the *lifetime of the avalanche*.

(4) Once the probability, Eq. (1), was calculated t_s times for each pinning center, the external field was increased.

(5) We repeated steps 1 to 4 until we obtain 1000 avalanches.

The evolution of the vortex distribution in our simulations could be roughly visualized in the following way. As the external field is increased from zero, the average slope of the flux distribution profile starts to increase from zero, as j does for the average pinning center. Eventually, the condition $j = j_c$ is reached for all pinning centers, so the profile is perfectly linear, and is called critical. As the external field is further increased, j increased above j_c and, after a certain value of H (in which a "supercritical" average profile is reached), the slope eventually decreases to $j = j_c$ due to the exit of vortices through the surface opposite to the one of the applied field, defining an avalanche. The "critical–supercritical–critical" sequence repeats again and again as the external field is increased.

It should be pointed out that, though our algorithm seems very similar to the ones presented by Bak et al. [3], there is an important difference which, at least for superconductors, turns our approach more realistic: in our case, the motion of vortices depends exponentially on some activation barrier which is related with a critical parameter j_c (analogous to the parameter presented by Bak et al. [3]) and with the microscopic state of the system. However, our probability choice does not take into account thermal activation when $j < j_c$. If it did the relaxation effects would never allow the establishment of a steady critical state, though this constitutes a limitation of our model it is the simplest practical way to define the avalanche size, and to study the dynamic formation of the critical state after a field perturbation in our conditions.

3. Results and discussion

Figs. 1 and 2 show the distributions of avalanche sizes $D(s)$ and lifetimes $D(t)$ for different values of t_s for a system with $\eta = 1$. To obtain these results we made two restrictions in our program. First, we added only *one* vortex at the superconductor boundary when the field was increased. Second, we defined the critical current as proportional to a difference of just *one* vortex between two neighboring pinning centers. These last restrictions reduced our time of simulations but did not change our general conclusions. The system shows two qualitatively different behaviors. The first one is for $t_s < 220$ m.c.s., for which the critical state is never reached and we cannot asses the existence of avalanches or criticality and hence it is not represented in the figures. The second one appears for $t_s \approx 220$ m.c.s. and displays a power law behavior of $D(s)$ and $D(t)$ showing a good coincidence with the predictions of the SOC theory. It is interesting to note that the exponent in $D(s)$ and $D(t)$ increases for larger response times, giving a sharp distribution of avalanches around $s = 1$ or $t = 1000$ m.c.s. as t_s becomes larger.

t_s can be regarded as the time the system has to reorganize itself after the action of some perturbation, so we will call it "arrangement time". If it is too short, we do not observe the formation of the critical state. Experimentally, this behavior is observed if the applied field is increased very rapidly [6]. For intermediate times, the system displays avalanches in order to maintain its organization. Finally, for infinite times all the avalanches will have the same size, i.e. every vortex added to the system will exit.

In Fig. 3 we represent the distributions of avalanche sizes corresponding to a system with $\eta = 0$ 1 for different arrangement times under the same restrictions as in Figs. 1 and 2. From the picture we can see that the time needed to reach the critical state is lower than in the case represented in Figs. 1 and 2. (A similar conclusion is reached studying the distributions of avalanche times.) This means that the system "answers" quickly the external perturbation for lower values of η, i.e. for lower U_0 and higher temperatures.

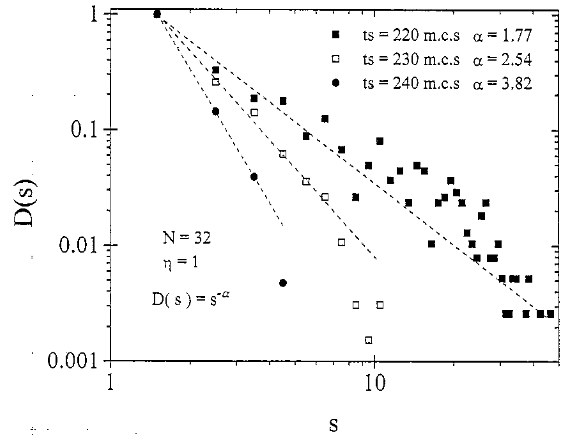

Fig. 2. Distribution of avalanche lifetimes for $\eta = 1$.

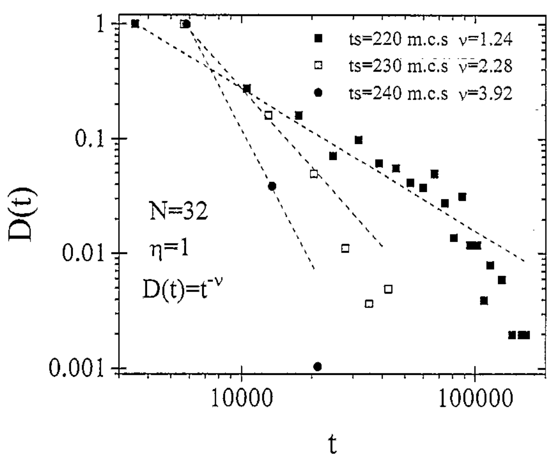

Fig. 1. Distribution of avalanche sizes for $\eta = 1$.

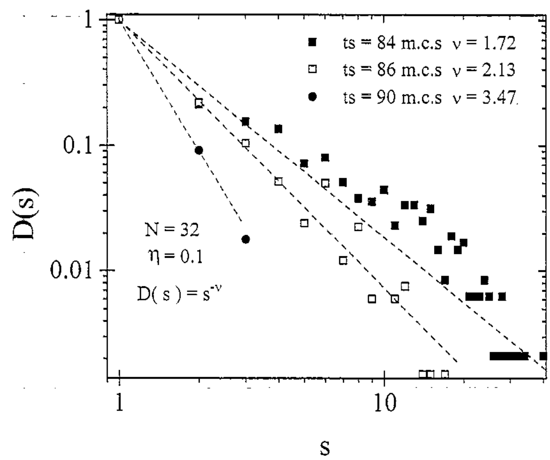

Fig. 3. Distribution of avalanche sizes for $\eta = 0.1$.

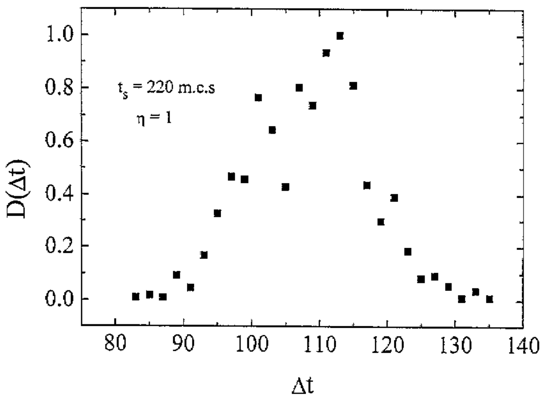

Fig. 4. Distribution of times between avalanches for $\eta = 1$.

These results suggest that the presence or not of power law distributions of avalanche sizes and lifetimes is determined by two causes: the rate of variation of the applied field (as was experimentally demonstrated in Ref. [6]) and the relation between the pinning and the thermal energies in the system, which seems to be consistent with recent experiments [8]. Then, one might interpret them in the following way: if the system is in the vortex glass regime and the perturbation (field variation) is slow enough, a critical state appears through the superconductor and it displays avalanches distributed following a power law. It should be clear that this state is minimally stable in the same sense of the one-dimensional cellular automaton of Bak et al. [13].

Finally, in Fig. 4 we plotted the distribution of times between avalanches for $t_s = 220$ m.c.s. and $\eta = 1$, which is a sharp function centered approximately at half the arrangement time. This result is not in agreement with the predictions of the SOC theory, but in our opinion it is a consequence of the assumption of periodical penetration of vortices into the superconductor.

4. Conclusions

We have reported Monte Carlo simulations in one-dimensional superconductors with a periodic distribution of pinning centers. We studied the system in the presence of a varying applied field, and we gave elements to state that, if the vortices are pinned in vortex-glass-like barriers and if the field is varied slowly enough, the system exhibits scale invariance for almost two decades as expected from the self-organized criticality theory and as real experiments show.

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References

[1] P.G. de Gennes, Superconductivity of Metals and Alloys, Benjamin, New York, 1966.
[2] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381.
[3] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. A 38 (1988) 364.
[4] V.M. Vinokur, M.V. Feigelman, V.B. Geshkenbein, Phys. Rev. Lett. 67 (1991) 915.
[5] Z. Wang, D. Shi, Phys. Rev. B 48 (1993) 4208.
[6] Z. Wang, D. Shi, Phys. Rev. B 48 (1993) 9782.
[7] S. Field, J. Witt, F. Nori, Phys. Rev. Lett. 74 (1995) 1206.
[8] G.A. Held, D.H. Solina, D.T. Kane, W.J. Haag, P.M. Horn, G. Grinstein, Phys. Rev. Lett. 65 (1990) 1120.
[9] E.R. Nowak, O.W. Taylor, L. Liu, H.M. Jaeger, T.J. Selinger, (1996), in press.
[10] L.P. Kadanoff, S.R. Nagel, L. Wu, S. Shou, Phys. Rev. A 39 (1989) 6524.
[11] O. Geoffroy, J.L. Porteseil, J. Magn. Magn. Mater. 97 (1991) 198.
[12] B. Drossel, F. Schwalb, Phys. Rev. Lett. 69 (1992) 1629.
[13] O. Pla, F. Nori, Phys. Rev. Lett. 67 (1991) 919.
[14] D.S. Fisher, M.P.A. Fisher, D.A. Huse, Phys. Rev. B 43 (1991) 130.
[15] C.W. Hagen, R. Griessen, in: Studies of High-T_c Superconductors (VIII), Nova Science Publishers, 1989.
[16] R. Mulet, L. Flores, E. Altshuler, Phys. Stat. Sol. (b) 182 (1994) K31.