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Universality of vortex avalanches in a type II superconductor with periodic pinning

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Abstract

In this work the robustness of a simple cellular automaton developed by Bassler and Paczuski (Phys. Rev. Lett. 81 (1998) 3761) to describe the critical state in type-II superconductors is studied. Two different configurations of pinning centers are introduced and a new universality class is found. The numerical values of the critical exponents were calculated following two scaling techniques to ensure the validity of our results. © 2000 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

In type-II superconductors, if the external magnetic field is greater than the first critical field H_{C1} , vortices penetrate the material. These vortices move freely in ideal superconductors if an electrical current is applied, destroying superconductivity [1]. However, in real materials, the existence of pinning centers prevent vortex motion if the current is less than a certain value, called critical current.

Bean in 1966 [2] proposed that the distribution of magnetic field inside a real sample, when the external magnetic field increases over H_{C1} , can be represented by a linear profile in an $H(x)$ graph, where the slope corresponds to the critical current density.

In 1989, Bak et al. [3], in an attempt to explain the behavior of many dynamical systems, developed the concept of self-organized criticality (SOC). In their approach, after long time of avalanches evolution, those systems reach a complex state which exhibit a power law in their avalanche size distributions and in their avalanche duration distributions, and $f^{-\beta}$ noise. Sandpiles have become a paradigm of these systems, and

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their closed similitude with the critical state of superconductors [4] quickly called the attention of researchers in the field [5–7].

In fact, Field et al. in 1996 [8] experimentally observed SOC in vortex avalanches at the inner wall of a low- T_c superconducting hollow cylinder submitted to slowly ramped axial magnetic fields. In a similar experiment performed on a low T_c thin film ring, Nowak et al. [9] observed either potential or peaked distributions of vortex avalanche size, depending on temperature. Finally, in a recent experiment, Mailly and Ethiehn [10] measured the internal vortex avalanches in a low- T_c film submitted to an increasing magnetic field, and found power distributions of avalanche size only for temperatures higher than 3 K.

On the other hand, many simulations had been performed on vortex avalanches. In 1994, Richardson et al. [11] developed a molecular dynamic model showing the discrete evolution of the magnetic profiles inside the superconductors. Also using molecular dynamic techniques, Olson et al. [12] tried to mimic Field's experiment. They found power distributions of avalanche sizes for high densities of pinning centers. However, none of these simulations was able to estimate correctly the critical exponents involved in the SOC theory for this system because of strongly finite size effects.

More recently, Bassler and Paczuski [13] proposed a simple cellular automata to model the dynamics of magnetic flux motion at a length scale larger than the range of vortex interaction for a system with a random distribution of pinning centers. They obtained, using finite size scaling techniques, the existence of critical exponents for a wide distribution of parameters in their simulation, and hence, of SOC. In this work, we explore the robustness of this cellular automaton varying the distribution and depth of the pinning centers.

In the next section we explain the Bassler and Paczuski model. In Section 3 we present and discuss the results obtained using two novel pinning centers configurations: periodic distributions and combined periodic plus random distributions. Finally the conclusions are given.

2. Model

The Bassler and Paczuski cellular automaton [13] is a two-dimensional hexagonal lattice where each site is occupied by $m(x)$ vortices. Vortices in site x can move towards site y only if the force acting on them, in that direction, is greater than zero. This force is calculated using the following formula:

$$F = V(y) - V(x) + (m(x) - m(y) - 1) + r(m(x_1) + m(x_2) - m(y_1) - m(y_2)) , \quad (1)$$

where x_1 , x_2 and y_1 , y_2 are the other nearest neighbors of x and y , respectively, and $V(x)$ and $V(y)$ are the strengths of the pinning centers at those sites. If the distributions of pinning centers are random, $V(x) = p$, with a probability q , and $V(x) = 0$ with a probability $1 - q$. The parameter r (where $r < 1$) characterizes the long distance action

of the next-nearest neighbours. If there is more than one unstable direction, one of them is chosen at random. All lattice sites are updated in parallel, and at each site only one vortex can move on a particular update. Periodic boundary conditions are applied to the top and the bottom of the lattice. The vortices that reach the right edge of the system are removed, while they are not allowed to abandon the system through its left edge.

An avalanche begins by randomly choosing a site at the left edge of the system and adding one vortex to it. It continues with the consecutive update of the lattice sites until no more unstable sites persist. Once the lattice is again stable, another vortex is added. The avalanche size is defined as the number of topplings corresponding to the addition of one vortex while the avalanche duration is defined as the number of updatings necessary to complete one avalanche.

To characterize the system, in analogy with other SOC models, Bassler and Paczuski [13] proposed and proved the following scaling ansatz for the probability distribution of avalanches sizes (2), and avalanches durations (3):

$$P(s, L) = s^{-\tau} g(s/L^D), \quad (2)$$

$$P(t, L) = t^{-\tau_t} g(t/L^z) \quad (3)$$

and obtained the following set of scaling exponents for a wide range of parameters values in the model $\tau = 1.63 \pm 0.02$, $D = 2.7 \pm 0.1$, $\tau_t = 2.13 \pm 0.08$ and $z = 1.5 \pm 0.1$ which are related by the scaling relations

$$\tau(2D - 1) = 1, \quad (4)$$

$$D(\tau - 1) = z(\tau_t - 1). \quad (5)$$

In our work, to ensure the validity of our results, we also used the following scaling ansatz [14] for the avalanches sizes and avalanches durations, respectively:

$$P(s, L) = L^{-\beta} f(s/L^D), \quad (6)$$

$$P(t, L) = L^{-\omega} f(t/L^z). \quad (7)$$

Therefore, two new scaling exponents characterize our system β and ω . They are related to the previous ones by $\beta = \tau D$ and $\omega = \tau_t z$, and satisfy the following scaling relations:

$$\beta = 2D - 1 \quad (8)$$

and

$$\omega = \beta + z - D. \quad (9)$$

With this notation, the Bassler and Paczuski results [13] give $\beta=4.4$, $D=2.7$, $\omega=3.19$ and $z = 1.5$.

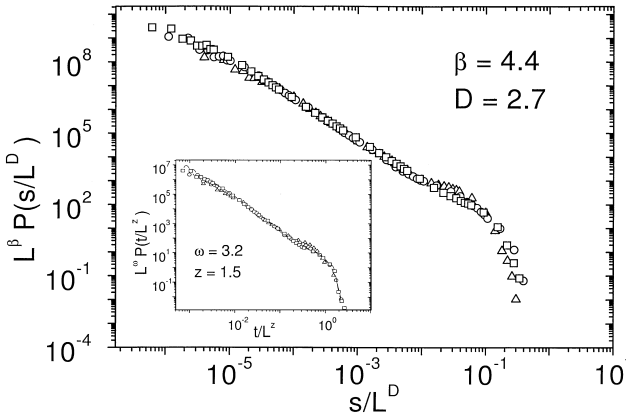


Fig. 1. Finite scaling plot using (6) $L = 100, 160, 200$ and $(r; p; q) = (0.1; 3; 0.1)$. Inset: finite scaling plot using (7).

3. Results and discussion

Following Bassler and Paczuski [13], we first assumed a random distribution of pinning centers, with parameters $(r, p, q) = (0.1; 3; 0.1)$. The collapse of our results for dimensions $L = 100, 160, 200$ using the scaling ansatz (6) and (7) can be seen in Fig. 1. The critical exponents for these conditions are in good agreement with Ref. [13]. Similar results were obtained for other parameters within the range already checked in [13].

Then, we tested the model using a combination of periodic and random pinning distributions. This kind of configuration introduces a new parameter to geometrically characterize the system, i.e. the distance between the periodic pinning centers a . In our simulations we used $r = 0.1$, and a constant density of random pinning centers $q = 0.1$. Like real irradiated superconducting materials, we used different pinning strengths p_1 and p_2 for periodic and random pinning centers, respectively. The periodic pinning centers were always chosen stronger than or equal to the random ones trying to mimic irradiation effects.

For the sets of parameters $(p_1; p_2; a) = (10; 1; 20), (10; 1; 10), (5; 1; 10), (10; 1; 4)$ and $(20; 1; 4)$ the critical exponents conserved the previously reported values of: $\tau = 1.63 \pm 0.04$, $\tau_t = 2.13 \pm 0.09$, $\beta = 4.4 \pm 0.1$, $D = 2.7 \pm 0.1$, $\omega = 3.2 \pm 0.2$ and $z = 1.5 \pm 0.1$ for different system dimensions, indicating the robustness of the system (see the two data collapse presented in Fig. 2).

We also performed simulations using only a periodic pinning configuration. It was demonstrated that the system also conserved the critical exponents for the sets of parameters $(p; a) = (5; 10), (5; 20), (5; 4), (1; 10)$ as is shown in Fig. 3. However, as the data collapse in Fig. 4 displays, using lower values of a and stronger pinning centers (for example: $(10, 4)$ and $(20, 4)$) caused a decrease in the exponents β, τ, τ_t and ω to $\beta = 3.2 \pm 0.1$, $\tau = 1.45 \pm 0.02$, $\tau_t = 1.7 \pm 0.08$ and $\omega = 2.6 \pm 0.2$, while $z = 1.6 \pm 0.1$ and $D = 2.2 \pm 0.1$, in good agreement with the scaling relations (4),(5),

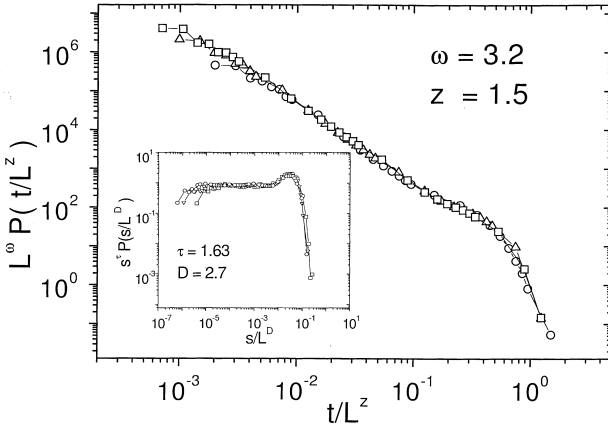


Fig. 2. Finite scaling plot using (7) $L = 100, 160, 200$ and $(r; p_1; p_2; a) = (0.1; 10; 1; 10)$. Inset: finite scaling plot using (2).

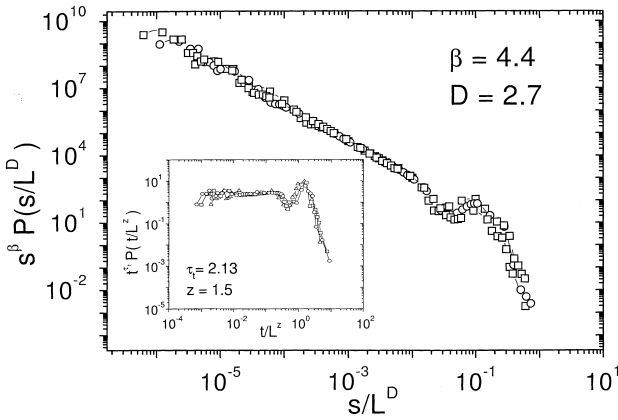


Fig. 3. Finite scaling plot using (6) $L = 100, 160, 200$ and $(p; a) = (5; 4)$. Inset: finite scaling plot using (3).

(8), and (9). This suggests the existence of a new universality class for dense and strong periodic pinning configurations.

To study the origin of this new universality class we made the calculation using a superposition of random and periodic pinning configurations with equal potentials $p_1 = p_2$. The scalings results, presented in Fig. 5, show that the systems with parameters $(p_1; p_2; a) = (10; 10; 4)$ conserved Bassler and Paczuski's [13] exponents values.

These results and those obtained using only a periodic configuration with the set of parameters $(p; a) = (5; 4)$ (see Fig. 3) proved that the new universality class appears due to the joint action of the strong periodic pinning centers and the negligible influence of the random pinning distribution. If in the system one of these conditions is absent, it will have a “normal” evolution.

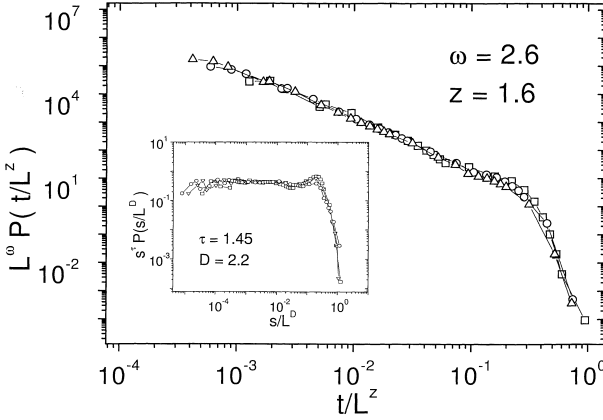


Fig. 4. Finite scaling plot using (7) $L = 100, 160, 200$ and $(p; a) = (10; 4)$. Inset: finite scaling plot using (2).

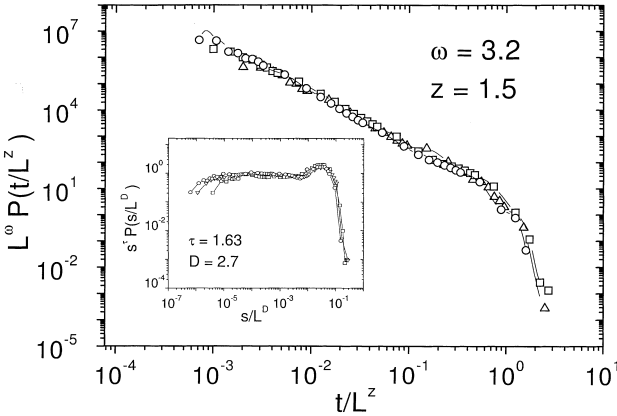


Fig. 5. Finite scaling plot using (7) $L = 100, 160, 200$ and $(r; p_1; p_2; a) = (0.1; 10; 10; 4)$. Inset: finite scaling plot using (2).

The existence of the new universality class could be explained based on the following qualitative argument. The presence of strong and correlated pinning, the negligible influence of the random pinning and the low values of a could produce “magnetic traps” [14] where vortices oscillate between neighbour pinning centers, increasing the number of bigger avalanches, and then reducing the characteristic exponents, τ, τ_l, β and ω .

Finally, an experiment is suggested to check the existence of this new universality class. It consists in performing an avalanche detection experiment similar to the one reported in Ref. [10], but before and after irradiating the sample with heavy ions in order to obtain arrays of periodic “strong” pinning centers as reported by Harada et al. [15]. If some relevant experimental parameters such as the magnetic field sweep rate

and the temperature are conveniently tuned, critical exponents close to those reported in [13] should be achieved before irradiation. After irradiation (under the same set of parameters) the combination of “weak” random pinning plus “strong” periodic pinning is expected to move the critical exponents towards the values reported here.

4. Conclusions

A cellular automaton model for vortex avalanches was analyzed in the presence of periodic-random and periodic distributions of pinning centers. The robustness of the model was proved for a wide range of parameters and the existence of a new universality class for strong and dense periodic distribution of pinning centers was shown. A qualitative argument to support the reasons for the appearance of this new universality class was given.

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