# Thermally activated avalanches in type-II superconductors

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Using a simple cellular automata with stochastic rules, we show the possible emergence of thermally activated avalanches (power law distributed) in type-II superconductors. Scaling relations between the exponents characterizing these distributions and those obtained from field-driven experiments are derived and proved through simulations. It is also shown that the conditions for the appearance of these avalanches are independent of the pinning mechanism. The relevance of our simulations for recently reported experimental results is also outlined.

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## I. INTRODUCTION

Magnetic flux penetrates type-II superconductors above a certain critical field $H_{c1}$ in the form of vortices. The interaction of these vortices with the pinning centers produces a magnetic-flux profile inside the superconductor with a slope proportional to the critical current density inside the sample, $j_c$, defined as the maximum current density that the material supports without dissipation, a situation that is accounted for by the so-called Bean's critical state model.[1] This picture, as de Gennes noted,[2] is very similar to the case of sandpiles, where a constant slope appears in the pile resulting from the competition between gravity and the friction between grains.

In 1987, Bak *et al.*[3,4] proposed a theory—now known as the self-organized criticality theory (SOC)—to explain the existence of self-similar structures in nature. Since then, SOC has been used to interpret the dynamics of many sizes of avalanches in sandpiles,[5] earthquakes,[6] evolution,[7] and other phenomena; see Ref. 8 for a general review.

The occurrence of self-organized criticality was soon researched also in superconductors where field-driven experiments have been designed[9–11] and many numerical simulations developed,[12–17] unfortunately without conclusive answers.

Superconductors differ from most systems exhibiting SOC by the relevant role played by temperature. The temperature causes relaxation of the critical state leading to a nearly logarithmic magnetization decay, $m(t)\sim \ln(t)$.[18] In the early 1990's many researchers tried to relate the role played by temperature to the existence of many sizes of avalanches in relaxation experiments.[19–21] However, in 1995 Bonabeau and Lederer[22,23] approximately solved the diffusion equation for the magnetic field inside a superconducting slab and demonstrated that (within the usual accessible time scales in the experiments) it is impossible to determine the existence of thermally activated avalanches by classical magnetic relaxation measurements, i.e., by the study of the decay of the mean value of the magnetization in the sample.[18,21]

In 1998 Aegerter[24] studied the magnetic relaxation of a single crystal of the HTSC compound Bi-2212, but instead of the usually measured mean value of $m(t)$,[18] he focused his attention on the fluctuations during the decay of the magnetization and showed evidences of power-law-distributed thermally activated avalanches.

In this work we develop a simple scenario enabling an account of the existence of these thermally activated avalanches. This does not mean we claim the existence of SOC during the relaxation of the magnetization. SOC is well defined only for a system in a marginal stationary state, which it is not the case for the vortex lattice in the presence of thermal activation. What we are claiming is that, because of the complex interaction between vortices, the pinning centers and the temperature, many sizes of avalanches of moving vortices may produce the relaxation of the critical state as previously determined in Ref. 24.

The remainder of the paper is organized as follows. In the next section we describe the cellular automata used in our simulations. In Sec. III we present and discuss numerical results. Then, Sec. IV is devoted to the study of the scaling relations between our distributions and those usually obtained in field-driven experiments. In Sec. V we outlined some conditions needed for the occurrence of many sizes of thermally activated avalanches and, finally, in Sec. VI conclusions are given.

## II. MODEL

While the use of ''real'' forces between vortices in molecular dynamics simulations[12,13] better resembles the experimental situation rather than simple cellular automata, they are by far more time-consuming and it is an important drawback of the method, especially when we are looking for critical exponents or when we introduce the effects of temperature on the system.

Recently, to avoid these problems, Bassler and Paczuski[14] introduced a simple cellular automata to study the behavior of the vortex lattice in type-II superconductors. This cellular automata avoids part of the relevant physics of the vortex lattice such as the variation of the pinning strength with the increasing field, the possible mismatch between the vortex lattice and the pinning centers, the elasticity of the vortex lattice, etc. However, it contains the interaction between vortices and pinning centers, and the long-range order of the vortex interaction, first by introducing parameter $r$ (see below), and then implicity assuming that each lattice cell contains more than one vortex. In addition, it is able to predict the self-organization of the lattice in a critical state characterized by power-law-distributed avalanches[14,15] and the irreversibility of magnetization. Then, aimed at describing the influence of temperature on this critical state, we adopt this model with some modifications.

The cellular automata consists of a two-dimensional 2D honeycomb lattice, where each site is characterized by the number of vortices on it, $m(x)$, and by its pinning strength, $V(x)$, equal to 0 with probability $p$, and to $q$ with probability $1-p$. The force acting on a vortex at site $x$ in the direction of site $y$ is calculated as

$$F_{x\rightarrow y}=-V(x)+V(y)+[m(x)-m(y)-1] +r[m(x_1)+m(x_2)-m(y_1)-m(y_2)], \qquad (1)$$

where $x_1$ and $x_2$ are the nearest neighbors of $x$ (other than $y$), while $y_1$ and $y_2$ are the nearest neighbors of $y$ (other than $x$) and $r$ is a measure of their contribution on the total force of the vortex $x$ $(0<r<1)$. A vortex in site $x$ moves to its neighbor site $y$ if the force acting on it in that direction is greater than zero. If the force in more than one direction is greater than zero, then one of them is chosen at random.[14–16]

To introduce the effect of temperature, we assumed that sites where the forces are lower than zero still have a probability of motion given by

$$P_{x\rightarrow y}\sim\exp[-U(j)/kT], \qquad (2)$$

where $k$ is the Boltzman's constant and $T$ is the temperature. The current, $j$, was locally calculated using the gradient of $m(x)$, and $U(j)$ represents different pinning barriers proposed in the literature: $U(j)=U_o j_c/j$, $U(j)=U_o\ln(j_c/j)$, and $U(j)=U_o(1-j/j_c)$.[25]

An avalanche starts by randomly choosing a lattice site, and calculating Eq. (2). If it is smaller than a random number the procedure is repeated, or else the vortex moves perturbing its neighbors. Then, the direction of motion of the new unstable vortices is calculated using Eq. (1). At this point, all the sites are updated in parallel until no more unstable sites persist. The avalanche size is defined as the number of topplings corresponding to the thermal activation of one vortex, while the avalanche duration is defined as the number of updatings necessary to complete one avalanche.

In all cases the procedure was repeated for $10^4$ Monte Carlo Steps (MCS) were one MCS was defined by the $L^2$ calculation of Eq. (2), and lattices up to $L=200$ were used. The initial configuration was obtained by slowly adding vortices to the system (at $T=0$) until a critical slope was reached.[14] The boundaries ''parallel to the net vortex motion'' were assumed periodic, while the other two were fixed to mimic the applied external field. All of the calculations presented in the paper were done with the following set of parameters: $(r,p,q)=(0.1, 5, 0.1)$ , but similar results were obtained using other values.[14]

The magnetization $M$ was calculated as the mean magnetic field inside the sample minus the external applied field,[17] i.e.,

$$M=\sum_{i=0}^{i=L} B(i)-H, \qquad (3)$$

where $H$ is the field, i.e., the number of vortices at the borders of the lattice.

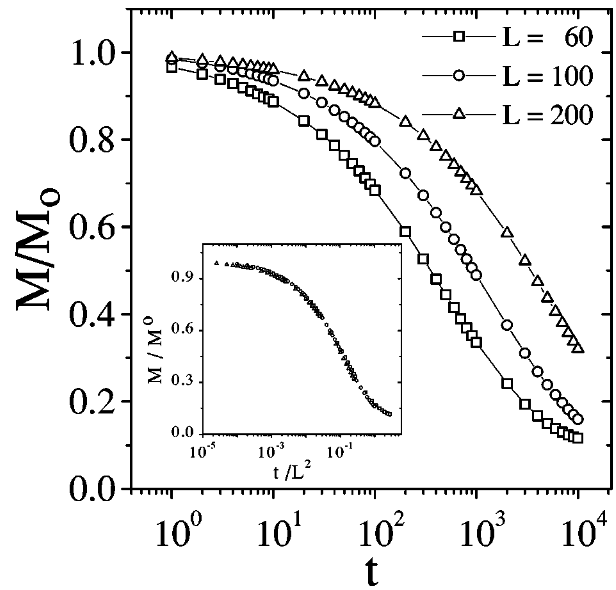


FIG. 1. Magnetic relaxation curves for systems of sizes $L=60$, 100, and 200. $U(j)\sim j_c/j$, $U_o/kT=10$. The inset shows the data collapse of the curves.

## III. NUMERICAL RESULTS

Figure 1 shows typical relaxation curves obtained for systems of different sizes using a vortex glasslike potential $U(j)\sim j_c/j$ (Ref. 25) and the algorithm described above. In Fig. 2 is represented the relaxation curve for a system with $T\sim\infty$. At this temperature, the thermal activation is so high that vortices would be continually jumping from their pinning centers, which allows the avalanchelike behavior previ-

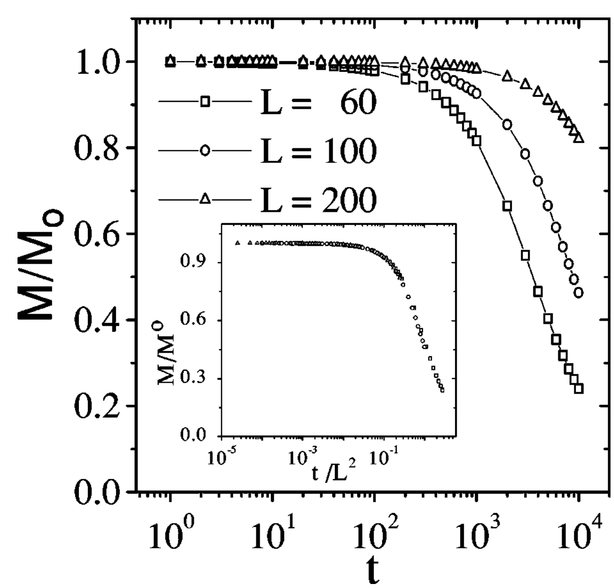


FIG. 2. Magnetic relaxation curves for systems of sizes $L=60$, 100, and 200. $U(j)\sim j_c/j$, $U_o/kT=0$. The inset shows the collapse of the curves.

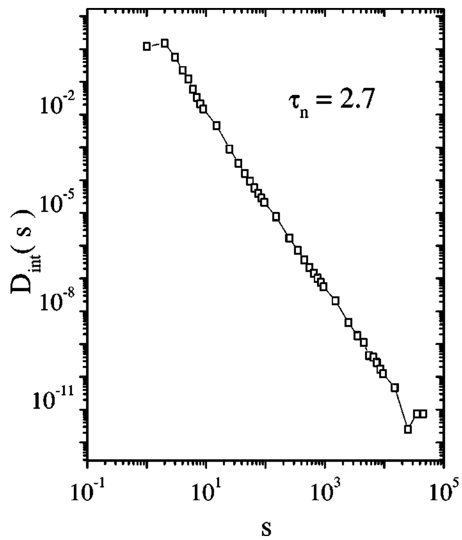


FIG. 3. Avalanche size distribution for $L=200$, $U(j)\sim(1-j/j_c)$, and $U_o/kT=10$.

ously explained to be disregarded (see Sec. V for a more careful discussion of this point), eliminating the calculation of Eq. (1) after a thermal jump.

In both figures (see also the inset) three regimes are present: a plateau, then a logarithmic relaxation, and finally another plateau due to finite-size effects. Only the time scales for these regimes are different, but this is irrelevant from an experimental point of view. So, as already noted before,[22,23] our results suggest that it is not possible to decide about the existence of thermally activated vortex avalanches from ‘‘simple thermodynamic magnetic’’ relaxation measurements. Other pinning potentials as well as different $Uo/kT$ relations were used[25] and no fundamental differences with the previous results were obtained.

Figures 3 and 4 represent the integrated (the meaning of this name will be clarified below) distribution of avalanche sizes, $D_{int}(s)$, and the integrated distribution of avalanche

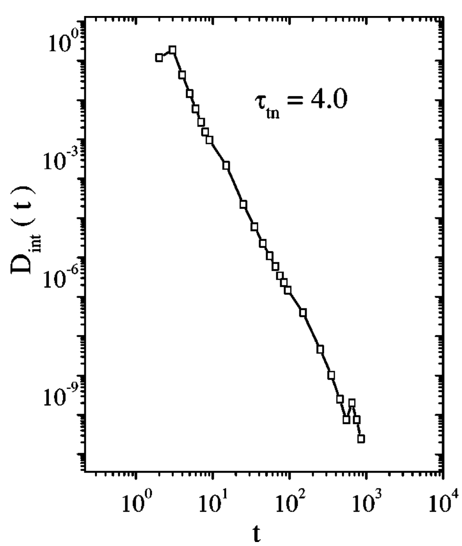


FIG. 4. Avalanche time distribution for $L=200$, $U(j)\sim(1-j/j_c)$, and $U_o/kT=10$.

times, $D_{int}(t)$, obtained using a classical Anderson-Kim potential, $U(j)=U_o(1-j/j_c)$,[26,25] for a system with $L=200$ and $U_o/kT=10$; as before, other pinning potentials were also used, resulting in a similar behavior. These distributions were obtained using the avalanche sizes and times (defined in Sec. II) obtained during all the relaxation processes.

As Figs. 3 and 4 clearly show, many sizes of avalanches emerge. It *does not* mean the system is critical; instead, it is relaxing from a critical state to its corresponding thermodynamic equilibrium. What we are showing is that this relaxation could proceed by means of many sizes of avalanches in accordance with recent experimental results.[24] However, somehow more surprisingly, we will show in the next section that the exponents characterizing these distributions are related through simple scaling relations to the exponents derived in the context of the SOC for systems in a critical state.[14,15]

Considering that $D_{int}(s)$ follows a power law,

$$D_{int}(s)\sim s^{-\tau_n}, \tag{4}$$

the estimated exponent from Fig. 3 was $\tau_n=2.70\pm0.1$ (different from the $\tau=1.63$ obtained in Refs. 14 and 15 for a field-driven experiment) and, assuming $D_{int}(t)\sim t^{-\tau_{tn}}$ for the integrated distribution of avalanche times, we obtained from Fig. 4, $\tau_{tn}=4.0\pm0.2$.

It is worth mentioning here that the exponent $\tau_n$ was also experimentally determined in Ref. 24 and reported as $\tau_n=2.0$, lower than our value. This divergence can be explained since Figs. 3 and 4 represent the distribution of avalanches obtained for all relaxation processes, i.e., starting at the critical state and finishing at equilibrium, a situation impossible to account for in real experimental situations.

We obtained different estimates for $\tau_n$ and $\tau_{tn}$ if, instead of the previous distributions, we used distributions of avalanche sizes and times obtained during the incomplete relaxation of the critical state. In fact, Fig. 5 represents five avalanche size distributions, $P(s)$, obtained for different time intervals of the relaxation curve, from the upper to the lower curve, $t=1-10$, $t=11-100$, $t=101-1000$, $t=1001-10\,000$, and $t=10\,001-100\,000$ MCS, which superposition corresponds to the full relaxation of the system (see Fig. 1). The straight line represents the integrated distribution of avalanche sizes, $D_{int}(t)$, obtained in Fig. 3, $\tau=2.7$. Then, from the figure we can conclude that different exponents can be predicted depending on the range of times measured. For short enough times, the exponent is lower than that associated with $D_{int}$, while for long times a peaked distribution is obtained with only very small avalanches.

Another source for discrepancies between our numerical estimates and experimental situations comes from the change of regimes of relaxation. In fact, there is not *a priori* justification to assume that many sizes of avalanches will dominate the relaxation process within all the ranges of $j$ and $T$, a situation that was thoroughly analyzed in Refs. 22 and 23 and is discussed in a different context in Sec. V.

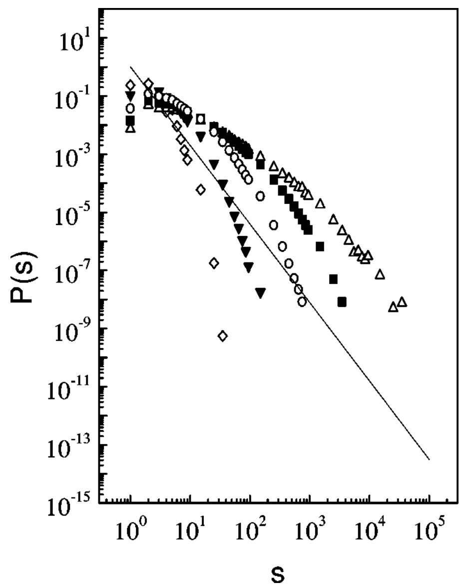


FIG. 5. Avalanche size distribution for $L=200$, $U(j)\sim j_c/j$, $U_o/kT=10$. From the upper to the lower curve: $t=1-10$, $t=11-100$, $t=101-1000$, $t=1001-10\,000$ and $t=10\,001-100\,000$ MCS. The straight line represents a power law with exponent 2.7.

## IV. SCALING RELATIONS

Instead of first presenting the derivation of our scaling relations, we prefer to start with a short review of some important scaling concepts of the theory of self-organized criticality.

Following the ideas of Bak *et al.*[3,4,8], those systems that behave as predicted by SOC show a distribution of avalanche sizes and times that follow power laws, i.e.,

$$P(s)\sim s^{-\tau} \tag{5}$$

and

$$P(t)\sim t^{-\tau_t}, \tag{6}$$

respectively. For systems not exactly in the critical state, these expressions transform into

$$P(s)\sim s^{-\tau}f(s/s_c) \tag{7}$$

and

$$P(t)\sim t^{-\tau_t}f(t/t_c), \tag{8}$$

where $s_c$ and $t_c$ reflect the departure of the system from criticality, $s_c\sim(j_c-j)^{-1/\sigma_1}$ and $t_c\sim(j_c-j)^{-1/\sigma_2}$, where $\sigma_1$ and $\sigma_2$ are new critical exponents and where the function $f(x)$ has the following properties: $f(x)\rightarrow constant$ if $x\rightarrow 0$ and $f(x)\rightarrow 0$ if $x\rightarrow\infty$ in order to recover the ''critical picture'' when $j\sim j_c$.

In finite-size systems $s_c$ and $t_c$ also reflect the effect of the sample dimensions through two new critical exponents $D$ and $z$. In fact, analogous to the theory of critical phenomena $s_c\sim L^D$ and $t_c\sim L^z$. Furthermore, the coherence length $\xi$ diverges at the critical state as

$$\xi\sim(j_c-j)^{-\nu}. \tag{9}$$

From the definitions of $s_c$, $t_c$, and Eq. (9) it is straightforward to show that $s_c\sim\xi^{1/\nu\sigma_1}$ and $t_c\sim\xi^{1/\nu\sigma_2}$. Moreover, since for a finite-size system at the critical state $\xi=L$, our first scaling relation takes the form

$$\frac{D}{z}=\frac{\sigma_1}{\sigma_2}. \tag{10}$$

As already discussed above, the integrated distributions of avalanche sizes and times, calculated in Sec. II, $D_{int}(s)$ and $D_{int}(t)$ result, since the system is relaxing from avalanches obtained for values of current densities ranging from $j_c$ to $j$. These distributions are different from those obtained in typical field-driven experiments or simulations, since the last are obtained ''in principle'' for a fixed value of current density $j_c$, which, indeed, determines the criticality of the system.

Then, it is natural to assume that $D_{int}(s)$ and $D_{int}(t)$ are related to the distributions obtained just at the critical state, $D(s)$ and $D(t)$, by the following formulas:

$$D_{int}(s)\sim\int_{j_c}^{0}s^{-\tau}f(s/s_c)dj \tag{11}$$

and

$$D_{int}(t)\sim\int_{j_c}^{0}s^{-\tau_t}f(t/t_c)dj, \tag{12}$$

which immediately explain the meaning of the label ''integrated'' used for these distributions.

Then substituting the definitions of $s_c$ and $t_c$ in Eqs. (11) and (12) and after a simple change of variables, we obtain the following expressions for the integrated distributions of avalanche sizes and times:

$$D_{int}(s)=s^{-\tau+\sigma_1}\int_0^{s(-j_c)^{1/\sigma_1}}\sigma_1 x^{\sigma_1-1}f(x)dx, \tag{13}$$

$$D_{int}(t)=s^{-\tau_t+\sigma_2}\int_0^{s(-j_c)^{1/\sigma_2}}\sigma_2 x^{\sigma_2-1}f(x)dx, \tag{14}$$

which prove that for large enough $s$ both integrals are constants, and there is not a cutoff length in the integrated distributions, a result already obtained in our simulations (see Figs. 3 and 4).

Also from Eqs. (13) and (14) and the definitions of $D_{int}(s)$ and $D_{int}(t)$ we can immediately obtain the following scaling relations:

$$\tau_n=\tau+\sigma_1, \tag{15}$$

$$\tau_{tn}=\tau_t+\sigma_2, \tag{16}$$

which in combination with Eq. (10) leads to

$$\frac{D}{z}=\frac{\tau_n-\tau}{\tau_{tn}-\tau_t}. \tag{17}$$

In this way expression (17) establishes a connection between the exponents obtained in field-driven experiments or simulations, $\tau,\tau_t,D,z$, and those from thermally activated avalanches $\tau_n, \tau_{tn}$. In fact, the results obtained in our simulations and those obtained in Refs. 14 and 15 hold the previous relation.

However, some points deserve further discussion. The power-law divergence of $s_c$, $t_c$, and $\xi$ are strictly valid close to the critical state, $j_c$. Far away from this state these divergencies no longer exactly hold; however, considering the good results obtained in the check of our calculations and scaling law (17), we believe that this last assumption is not relevant for the solution of the model. Also, scaling law (17) was obtained assuming the complete relaxation of the system, so it is difficult to be proved in real experiments.

## V. APPLICABILITY

Our previous picture assumes that a thermally activated vortex jump would affect its neighborhood, generating an instability that leads to a cascade of vortex jumps related to the vortex distribution into the sample.

However, the existence of a characteristic time for thermally activated phenomena $t_{th}=t_o \exp[U(j)/kT]$ is well known, representing the time a vortex spends at a pinning site before jumping due to thermal activation.[25]

This means that our model will be valid if these avalanches occur within times lower than $t_{th}$, i.e., the avalanches should develop fast enough to be mutually independent. This resembles the idea developed by Vespignani *et al.*[27] in the context of sandpile and forest fire models. They showed through simulations and mean-field considerations that one necessary condition for the occurrence of SOC, at least in these models, is the separation of time scales between the external excitation and the response of the system.

Then, as mentioned above, the maximum time that an avalanche persists is $t_c=t_{co}(1-j/j_c)^{-1/\sigma_2}$, where $t_{co}$ is the time a vortex spends moving from one site to another, and of course depends on the local current and flux density in the system. Considering that the vortices are separated by a distance $a$, the time they spend traveling this distance is

$$t_{co}=\frac{a}{v}, \tag{18}$$

where $v$ depends on the Lorentz force acting on the vortex $v=j\Phi_o/\eta$, and $a=(\Phi_o/B)^{1/2}$, which immediately gives the following dependence of $t_{co}$ with $j$ and $B$:

$$t_{co}=\frac{\eta}{j\sqrt{(\Phi_o B)}}. \tag{19}$$

In the critical state $B$ varies along the sample. This variation is, even in the presence of thermal activation, very well accounted for by the Bean model.[1,28] This means that, for a fully penetrated sample,

$$B(x)=\mu_o H-\mu_o j x, \tag{20}$$

where $H$ is the external field. Then, substituting Eqs. (19) and (20) in the definition of $t_c$, we obtain

$$t_c=\frac{\eta(1-j/j_c)^{-1/\sigma_2}}{j\Phi_o^{1/2}\sqrt{\mu_o H-\mu_o j x}}. \tag{21}$$

Now, to determine the regime of applicability of our model, we must verify under which conditions the inequality $t_c \ll t_{th}$ holds.

From the experimental point of view, the relevant avalanches to be detected measuring the fluctuations in the magnetization decay[24] are those starting at the border of the sample, since they are the ones that produce changes in $m$. Moreover, the avalanches starting at the border of the sample are also those with longer duration times since they have a larger area for spreading (remember that the critical state in a type-II superconductor is symmetric with respect to the center of the sample). Then, we may assume in Eq. (21) that $x=0$ and obtain the following inequality:

$$\frac{\eta(1-j/j_c)^{-1/\sigma_2}}{j\Phi_o^{1/2}\sqrt{\mu_o H}} \ll t_o \exp[U(j)/kT], \tag{22}$$

which can be written as

$$\frac{j^*}{j}\frac{1}{(1-j/j_c)^{1/\sigma_2}} \ll \exp[U(j)/kT], \tag{23}$$

where $j^*=(\Phi_o \eta/\sqrt{H})t_o$. Assuming, for example, $U(j)=U_o \ln(j_c/j)$,[25] the previous inequality takes the form

$$\frac{j^* j^{\alpha-1}}{j_c^{\alpha}}(1-j/j_c)^{-1/\sigma_2} \ll 1, \tag{24}$$

and $\alpha=U_o/kT$.

It is then straightforward to demonstrate that Eq. (24) holds under the following conditions: if $\alpha \gg 1$, $j$ must be much lower than $j_c$ ($j \ll j_c$). In the opposite case $j^* \ll j \ll j_c$. Similar expressions can be derived for the Anderson-Kim potentials and from potentials derived from the collective pinning theory.[25]

These conditions are consequences of the competition between the increase of $t_{th}$ when $j \to 0$ and the divergence of $t_c$ when $j \to 0$ and $j \to j_c$, see Eq. (21), and can be interpreted in the following way. Close to $j_c$ the avalanche durations are very high because the avalanche sizes become huge, so one always needs to be far from $j_c$, a situation often accounted for in high-temperature superconductors,[18] to assure that $t_{av} \ll t_{th}$. Particularly for $U_o \ll kT$, when thermally activated jumps become frequent ($t_{th}$ small), high enough currents ($j \gg j^*$) are also necessary to assure rapid vortex motion during the avalanche, and hence short avalanche time durations. From the experimental point of view these conditions should be taken with some caution. For example, since $j_c$ decays with temperature,[25] for $U_o \ll kT$, the range of current densities where thermally activated avalanches could appear is still narrower than that suggested by a simple inspection of the formula $j^* \ll j \ll j_c$, so we strongly recommend looking for these avalanches at low temperatures and in very disordered systems where $U_o \gg kT$.

In light of these results it is useful to come back to the experiment of Aegerter.[24] He found one critical exponent characterizing the avalanche size distribution during the relaxation of the magnetization and found that this exponent was independent of the temperature of the system. Neither of these results contradicts our model. Even when his critical exponent was 2.0 and our $\tau_n = 2.7$, Fig 5 indicates that small exponents are associated with short relaxation times in our model. This suggests that, if in the experiment of Ref. 24 the time window had been shifted to larger times, an exponent closer to our exponent would have been observed. This does not mean, of course, that such a shift can be trivially performed in practice.

In addition, he found during the relaxation one initial regime where avalanches are not power law distributed. While he explained that this was due to a transient period the system takes to reach the SOC, our results suggest a different explanation. During this period the system is still too close to the critical state $j \sim j_c$, and power-law avalanches are not yet developed since the thermally activated avalanches overlap each other. This explanation is consistent with the long time period associated with this transient period, and with the dependence of this time on the temperature. Experimentally Aegerter found that longer times are associated with higher temperatures and in fact, in our model higher temperatures imply the necessity of lower values of the relation $j/j_c$ to find power-law-distributed avalanches, and this means longer transient periods.

## VI. CONCLUSIONS

In conclusion, we developed a simple scenario to explain recently reported thermally activated avalanches power law distributed for type-II superconductors. We proved that the exponents associated with these distributions depend on the time interval of the measurement. We also proved that the exponents characterizing a distribution of thermally activated avalanches obtained during the whole relaxation experiment (i.e., from the critical to the equilibrium states) are related to those obtained in field-driven experiments by scaling relations, a situation also supported by our simulations. The conditions for the appearance of these avalanches were discussed, and it was also proved that, in a rough approximation, they do not depend on the pinning mechanism in the sample. All our theoretical predictions are consistent with known experimental results.

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