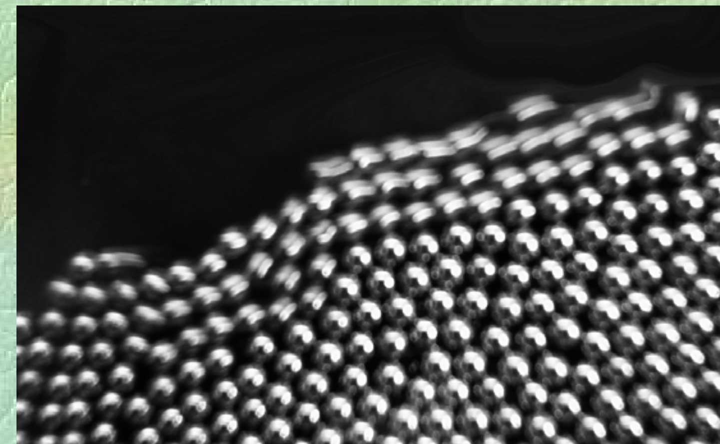
Avalanches in piles of grains

Osvanny Ramos

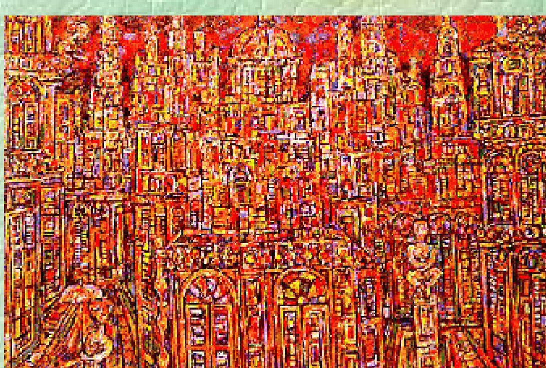


In collaboration with

Alfo J. Batista-Leyva

Ernesto Altshuler

Cuban complexity



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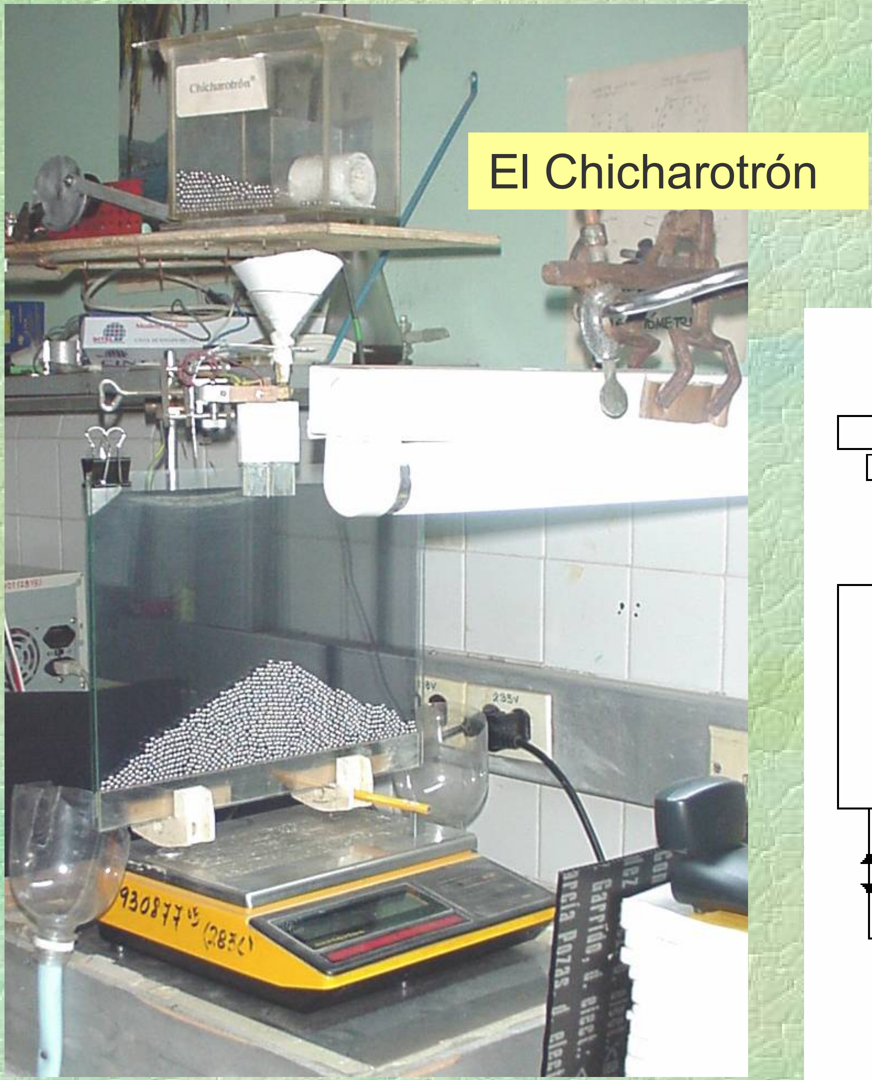
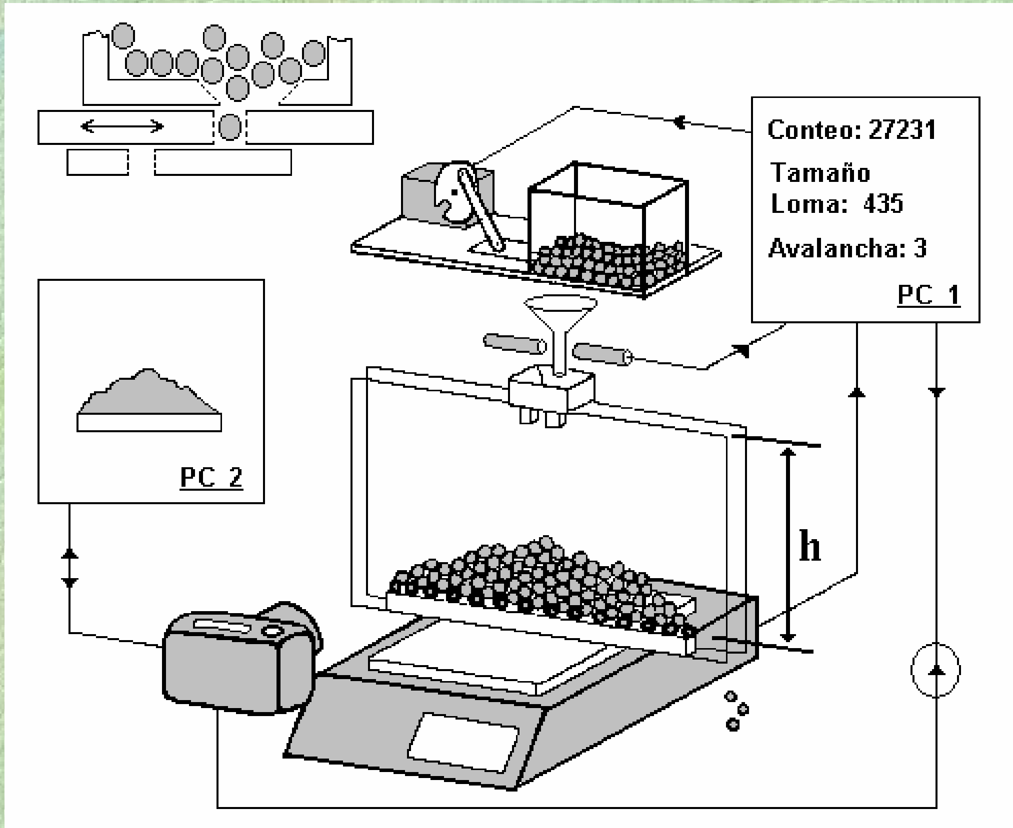
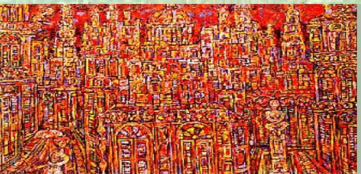
Experimental Setup



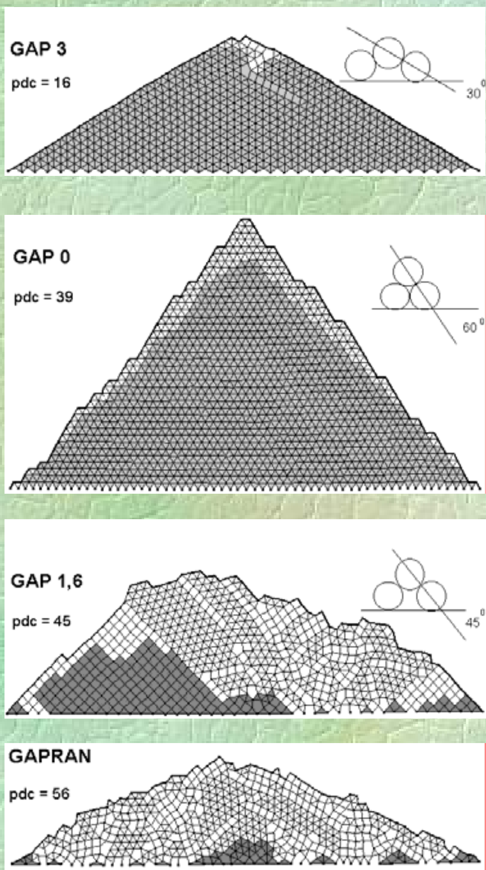




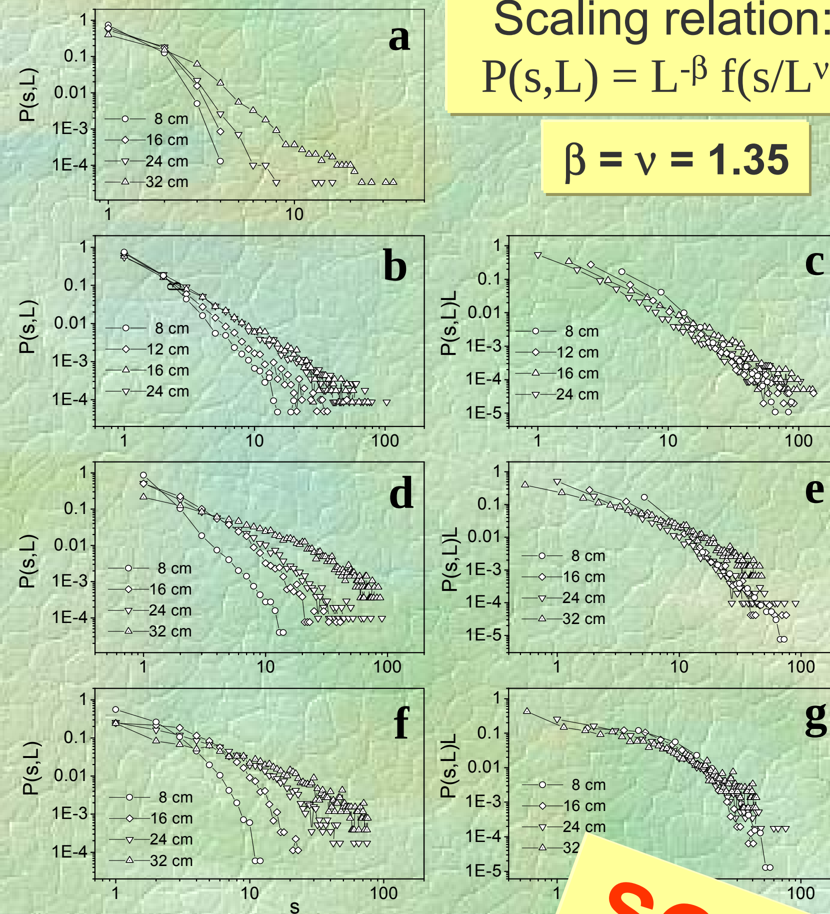
Influence of the **nature** of the base

Previous Results

Altshuler *et al*. *PRL* 86, 5490 (2001)



Increase of Inner disorder



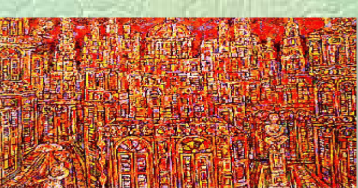
Scaling relation:

$$P(s,L) = L^{-\beta} f(s/L^{\nu})$$

$\beta = \nu = 1.35$

Increase of Scaling quality

SOC



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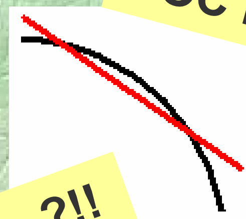
Questions

1 - Why

SOC logo ?!!



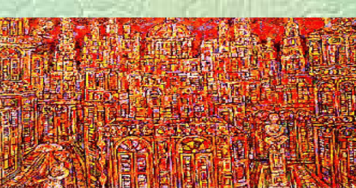
Power laws ?!!

?

- Should we expect power laws in *off the edge* avalanche distributions ?

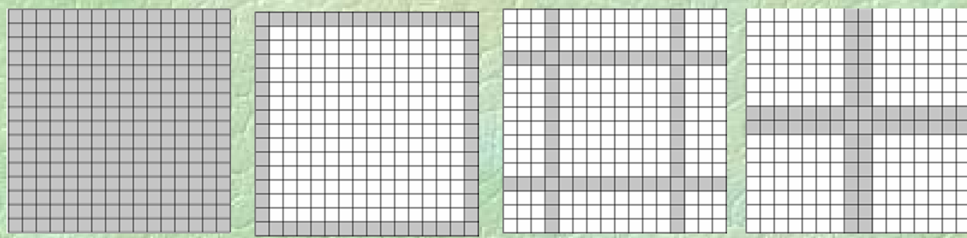
Relationship between *off the edge* and *internal* avalanches

2 - Can we **predict** avalanches in a SOC scenario ?

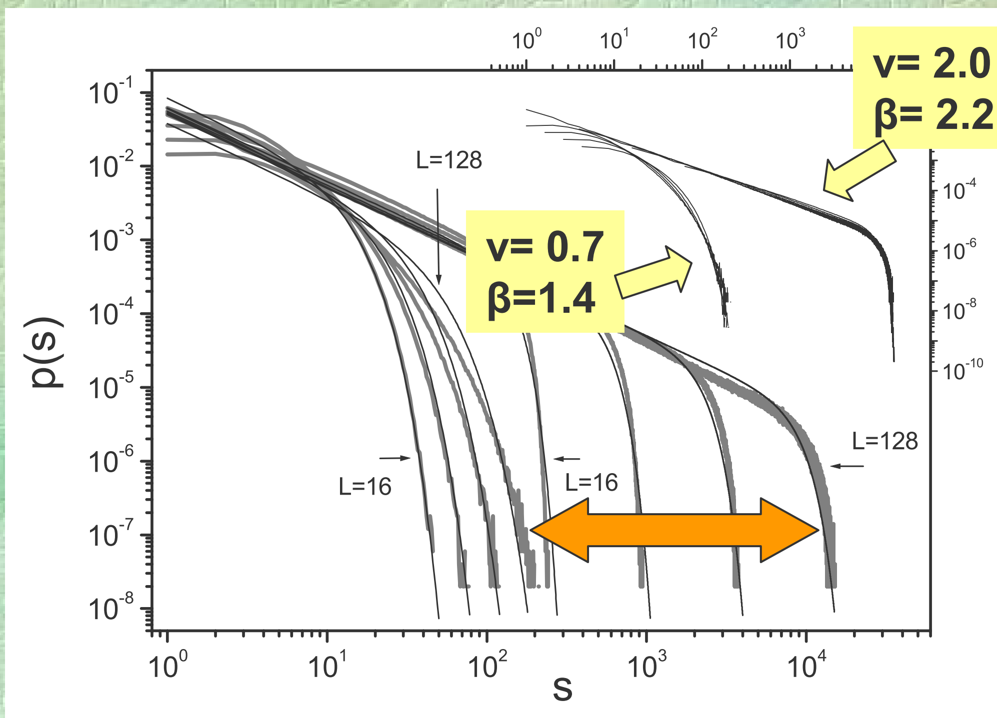


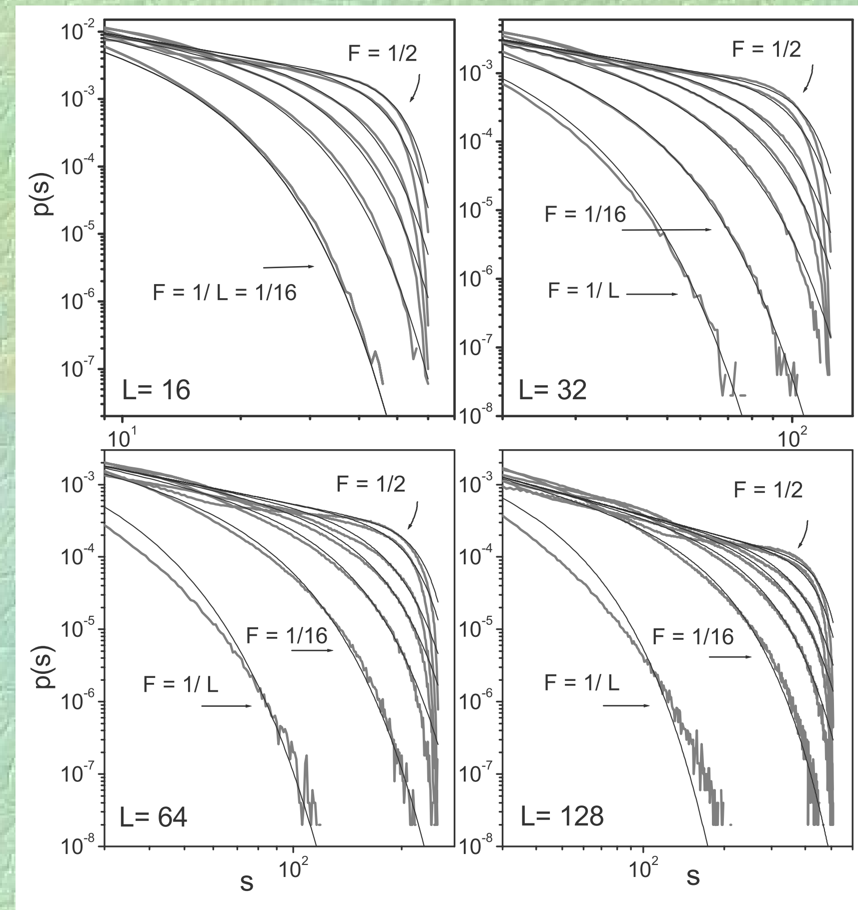
1- On the interpretation of “off the edge” avalanches

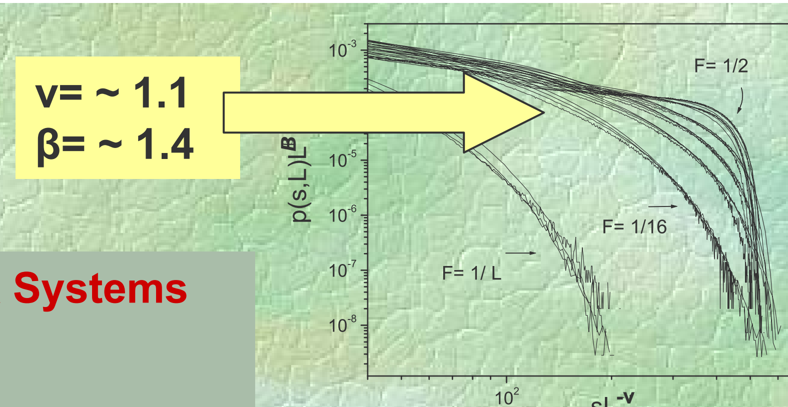
Simulations

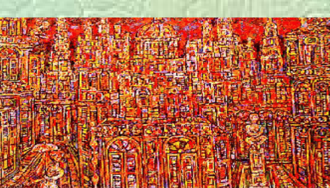


internal | off the edge F=1/L | F=1/4 | F=1/2









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1- On the interpretation of “off the edge” avalanches

Analytic

$$p(s) = p_0 \cdot s^{-1} \exp\left(-e1 \cdot \left(\frac{s}{N} \right)^{e2} \right)$$

where

$$e2 = \left(L / \sqrt{2R} \right)^2$$

$e1 = 8 \pm 1$ **for internal avalanches**

$e1 = A\, F^{\gamma} / L^{\xi}$ **for *F*-dependent sites**

$$A = 4.0 \pm 0.1$$

$$\gamma = -0.81 \pm 0.01$$

$$\xi = 0.28 \pm 0.01$$

In general

$$p(s) = p_0 \cdot s^{-\alpha} \exp\left(-e1 \cdot \left(\frac{s}{k \cdot L^d} \right)^{e2} \right)$$

Scaling relations

$$P(S) = p(s) L^{\beta} \qquad S = s \cdot L^{-v}$$

$$P(S) = p_0 \cdot S^{-\alpha} L^{\beta - \alpha v} \exp\left(-e1 \cdot \left(\frac{S L^{v-d}}{k} \right)^{e2} \right)$$

$$p_0 = L^{\eta} \qquad \eta = -0.37 \pm 0.01$$

Internal avalanches (d=2, k=1, e1=const, α=1)

v = d = 2 **β = α v** = 2

v= 2
β= 2

Constant *F* (d=1, k=4)

$$P(S) = S^{-\alpha} L^{\beta - \alpha v + \eta} \exp\left(-A\, F^{\gamma} (S/4)^{e2} L^{-\xi + e2(v-d)} \right)$$

v = d + ξ/e2 β = α v - η

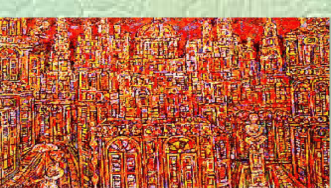
v= ~1.1
β= ~1.4

off the edge **(d=1, k=4, F=1/L)**

$$P(S) = S^{-\alpha} L^{\beta - \alpha v + \eta} \exp\left(-A\, (S/4)^{e2} L^{-\gamma - \xi + e2(v-d)} \right)$$

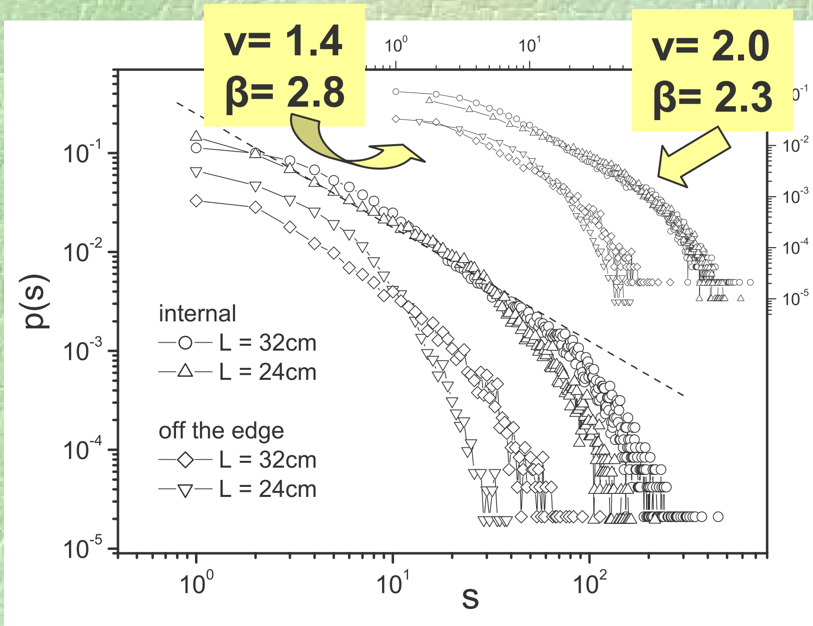
v = d + (ξ+ γ)/e2 β = α v - η

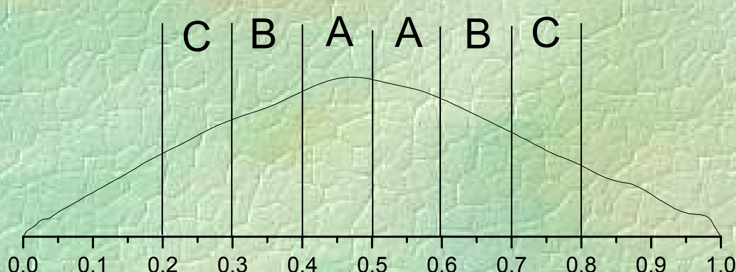
v= 0.7
β=1.4

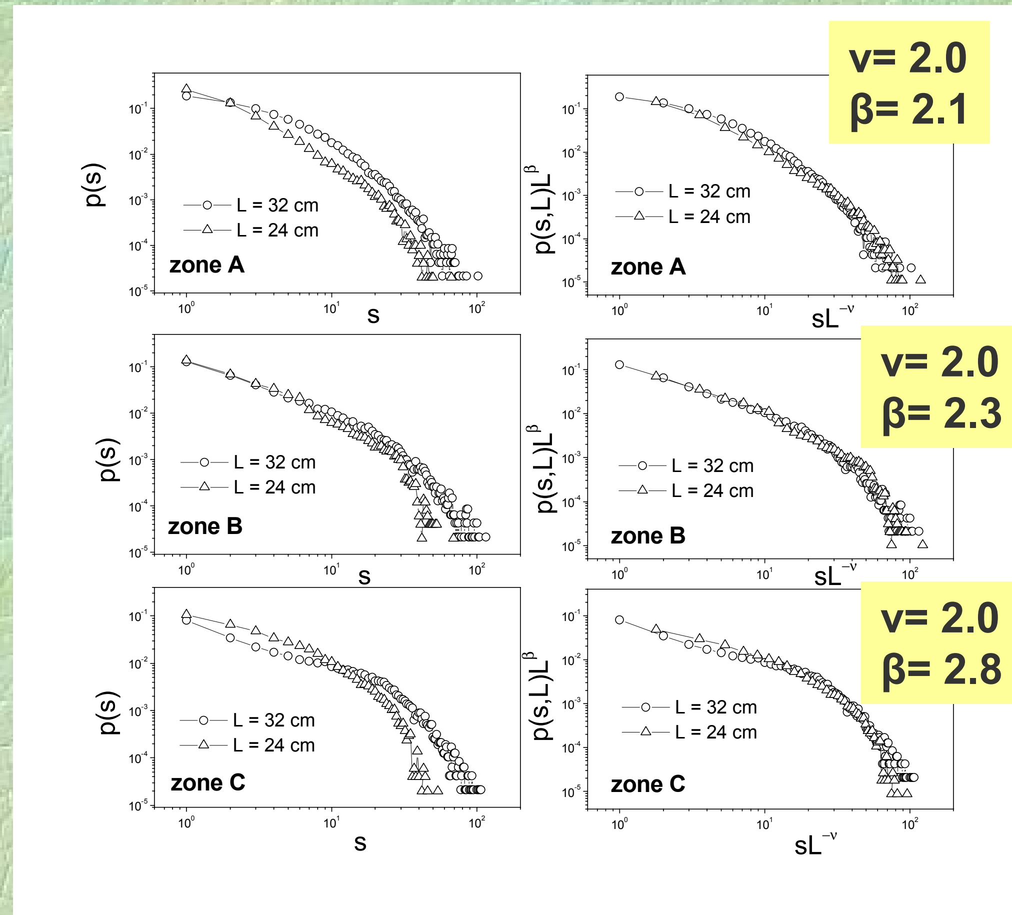


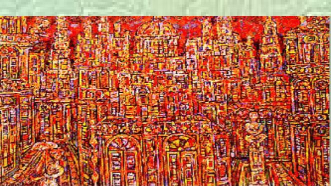
1- On the interpretation of “off the edge” avalanches

Experiment









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Experiment

Scaling relations

$$\langle s\rangle = \int_0^\infty sP(s,L)ds = \int_0^\infty sL^{-\beta} f(sL^{-\nu})ds$$

taking $x = sL^{-\nu}$

$$\langle s\rangle = L^{2\nu-\beta}\int_0^\infty xf(x)dx$$

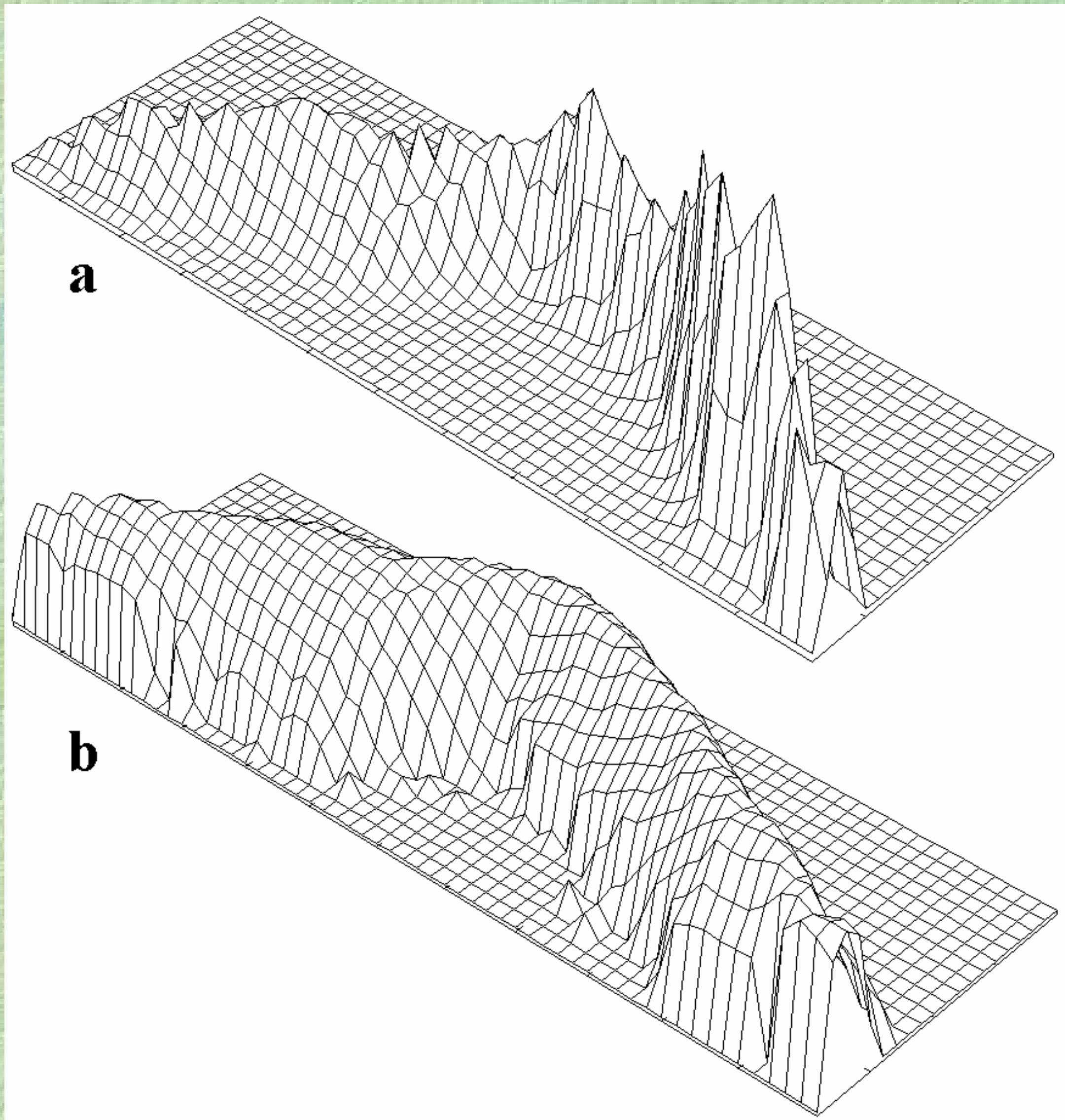
$$\langle s\rangle \propto L^{2\nu-\beta}$$

for *internal* avalanches

$\beta \approx \nu \approx d$ $\Rightarrow$ $\langle s\rangle \propto L^{d}$ $\Rightarrow$

for *off the edge* avalanches

$\langle s\rangle = 1$ $\Rightarrow$ $\beta = 2\nu$



Conclusions

1- The distributions of avalanches in proportional portions of the system behave similarly to the avalanches in the whole system.

2- The distributions of “off the edge” avalanches do not show power laws, but collapse when scaling relations are applied. The critical exponents do not correspond with their analogous for the internal avalanches, due to the former ones involve zones that are not a fixed proportion of the system size.

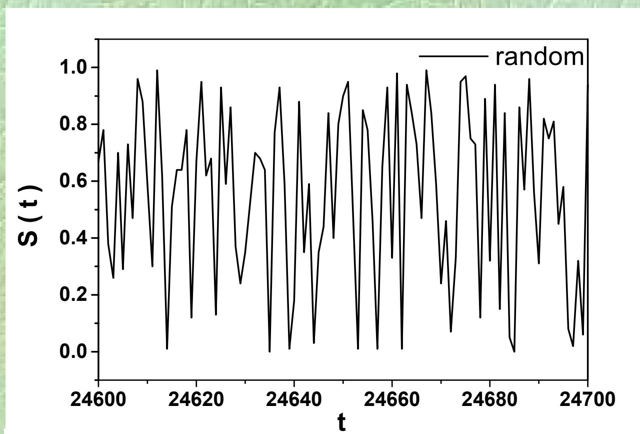
3- Scalings of “off the edge” avalanches can be taken as an indication of power law behavior of the internal avalanche size distribution in a model Sandpile

2- Trying to predict avalanches

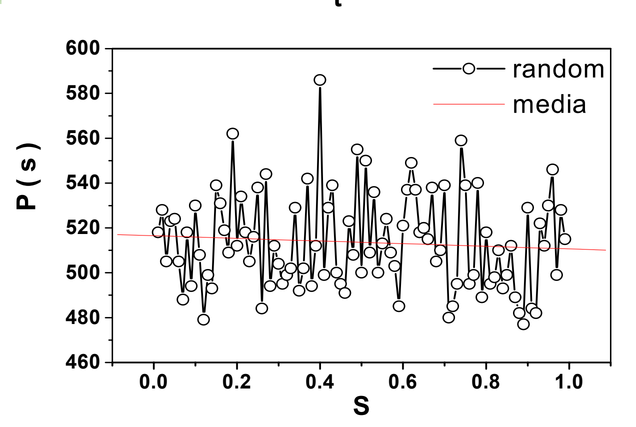
Correlation function

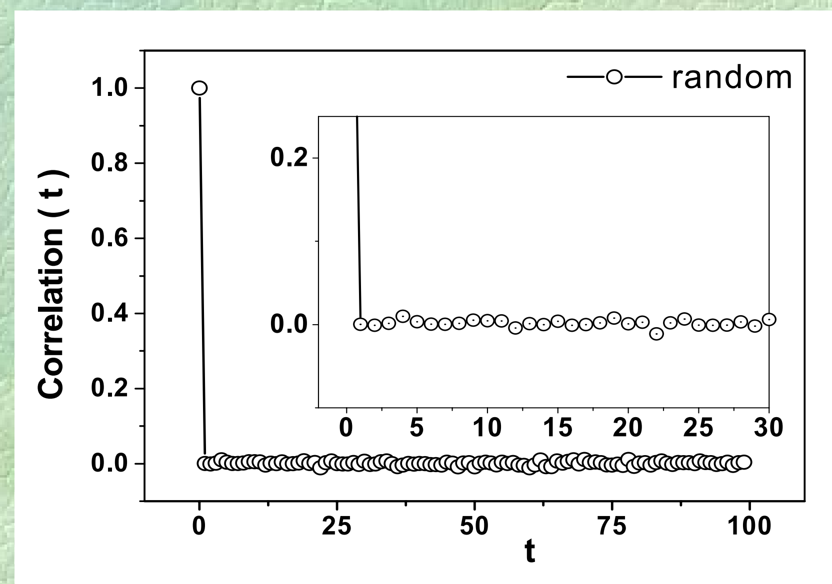
$$Fc(t) = \frac{Cov(x_1, x_2)}{\sqrt{Var(x_1) \cdot Var(x_2)}}$$

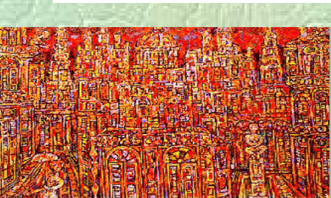
$$Fc(t) = \frac{\left(\frac{1}{T-\tau}\right)\sum_{\tau=1}^{T-\tau}\left(x_1(\tau)\cdot x_2(\tau+t)\right) - \left(\langle x_1(\tau)\rangle \cdot \langle x_2(\tau)\rangle\right)}{\left(\frac{1}{T-\tau-1}\right)\sqrt{\sum_{\tau=1}^{T-\tau}\left(x_1(\tau)-\langle x_1(\tau)\rangle\right)^2 \sum_{\tau=1}^{T-\tau}\left(x_2(\tau)-\langle x_2(\tau)\rangle\right)^2}}$$



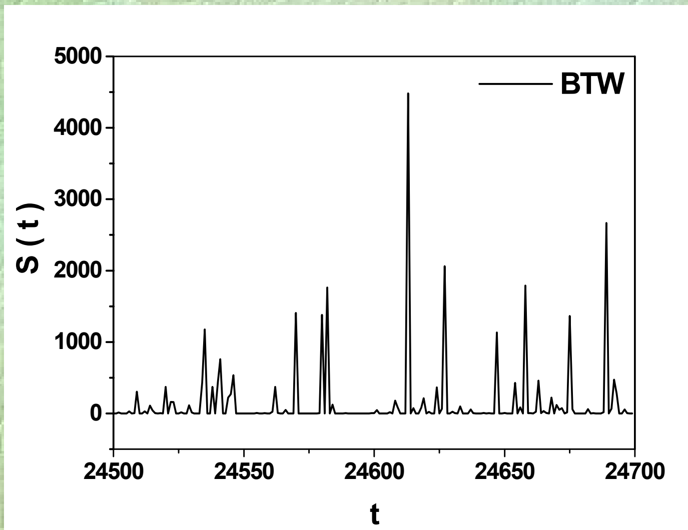
White Noise { 0 -1 (0.01) } ~ 50 000 counts

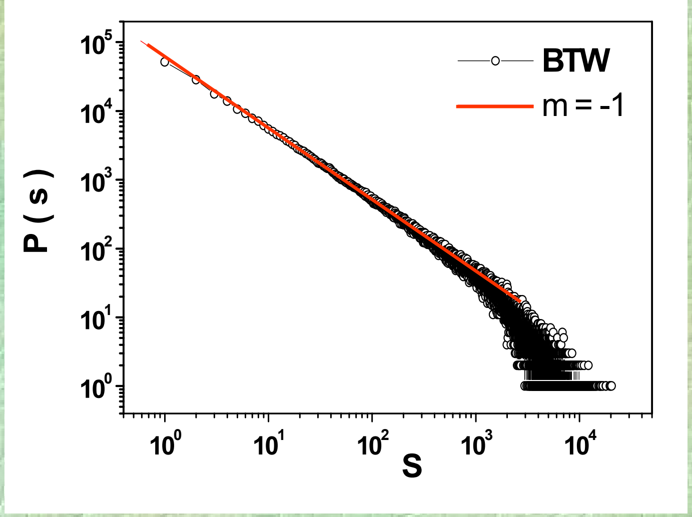






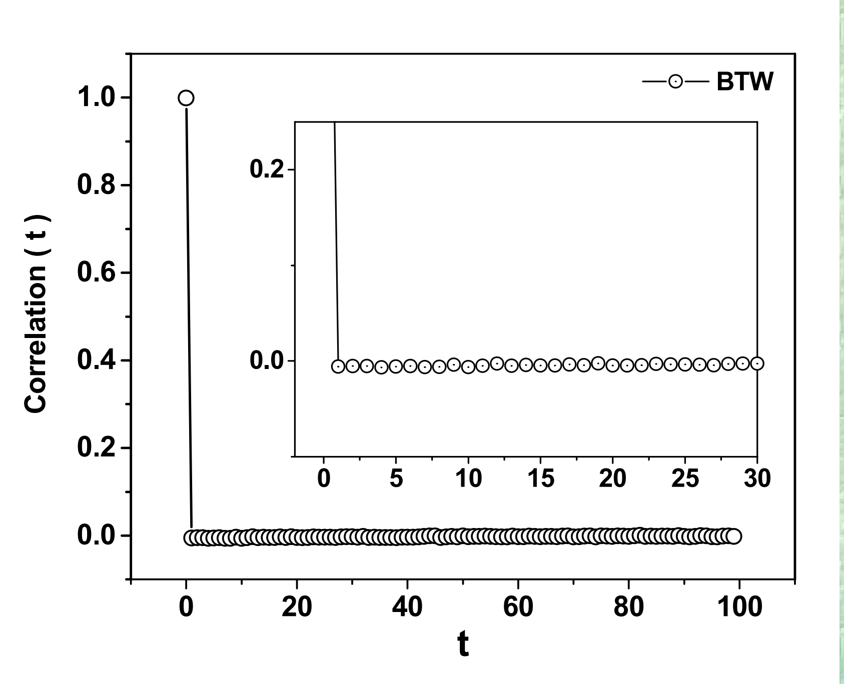
1- Trying to predict avalanches

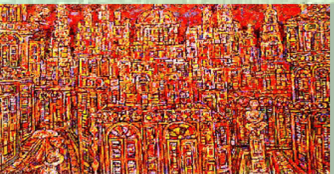


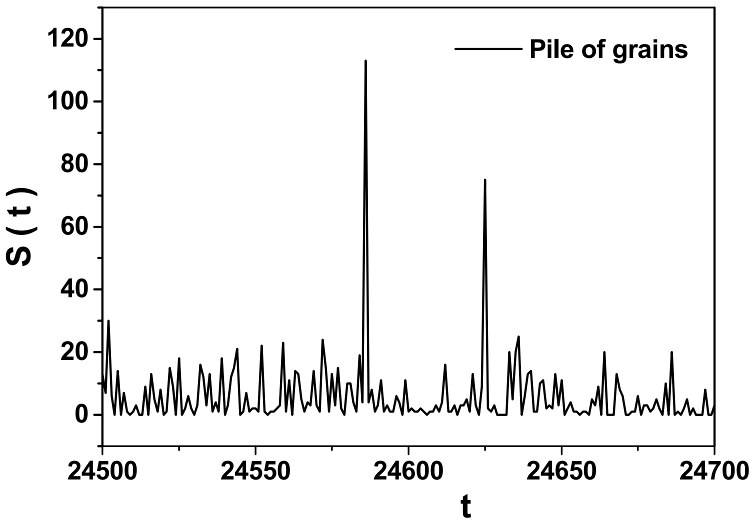


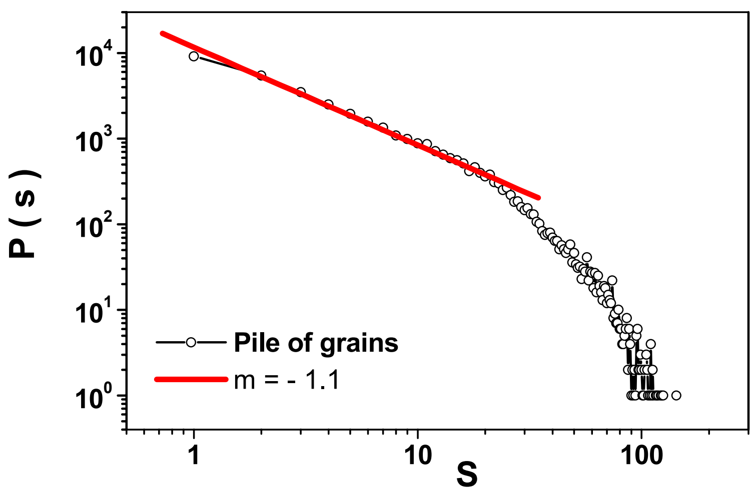
BTW

L = 64
10 ^{6} counts (topples)



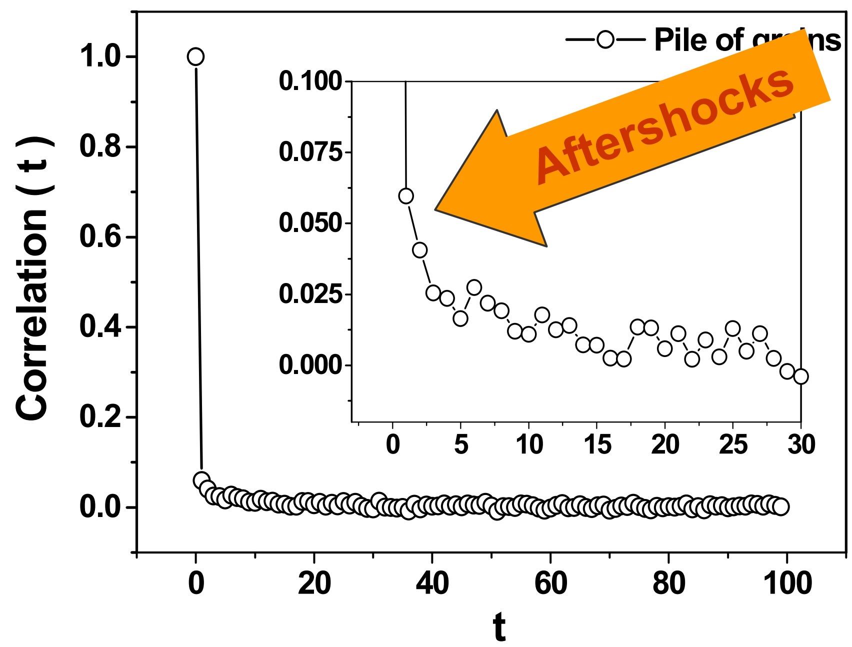






Pile of grains

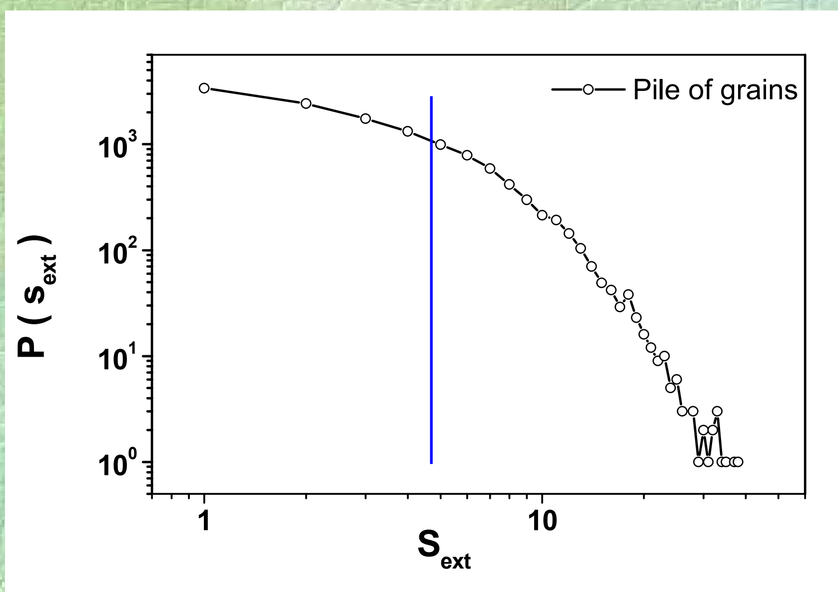
base length = 240 mm
50 000 counts
Internal avalanches

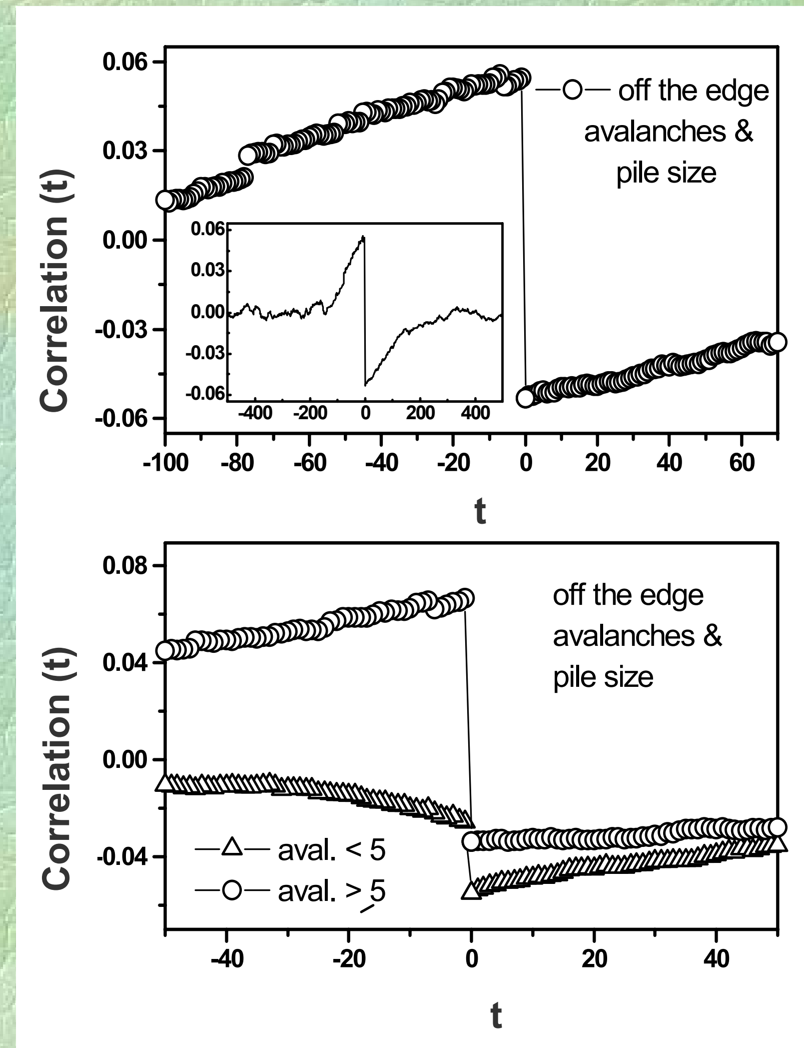


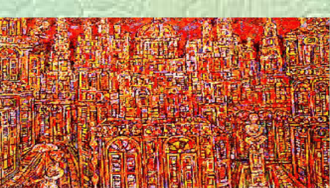
trivial correlation

Pile of grains

off the edge avalanches
&
pile size







When an avalanche is large?

Ley de Weber - Fechner: Sense Intensity ~ Log(stimuli Intensity)

Ley de Gutemberg - Richter: bM ~ Log(N)

where

M ~Log(Energy) magnitude

damage

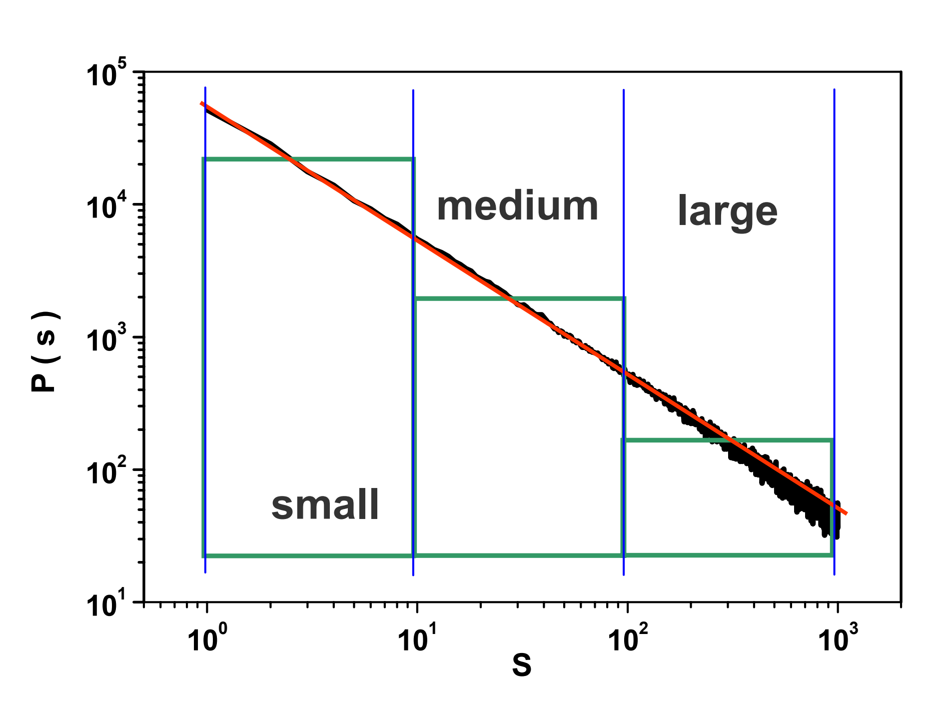
b ~ -1

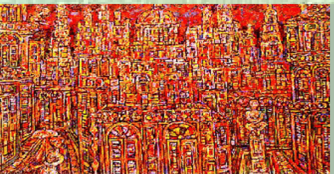
N = number of earthquakes with magnitude > M.

Then

Log [N (E)] ~ - Log E

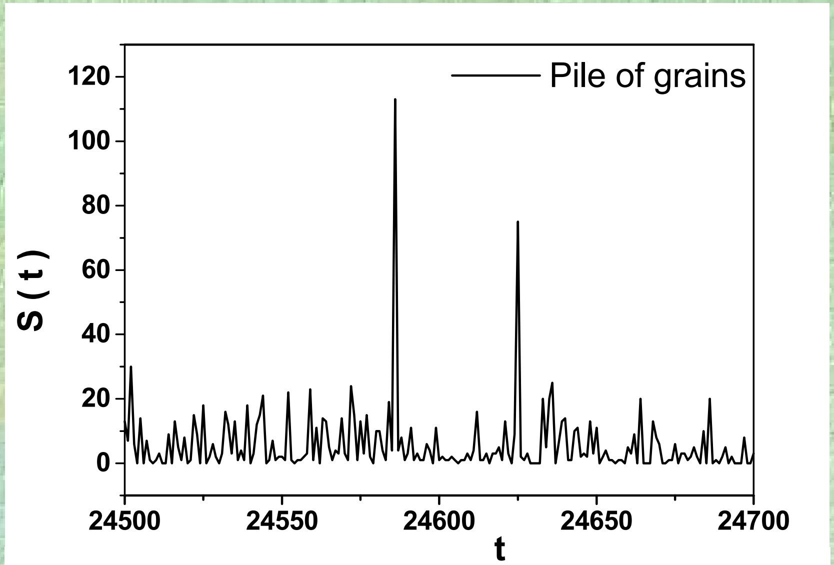
N (E) ~ E $^{-1}$

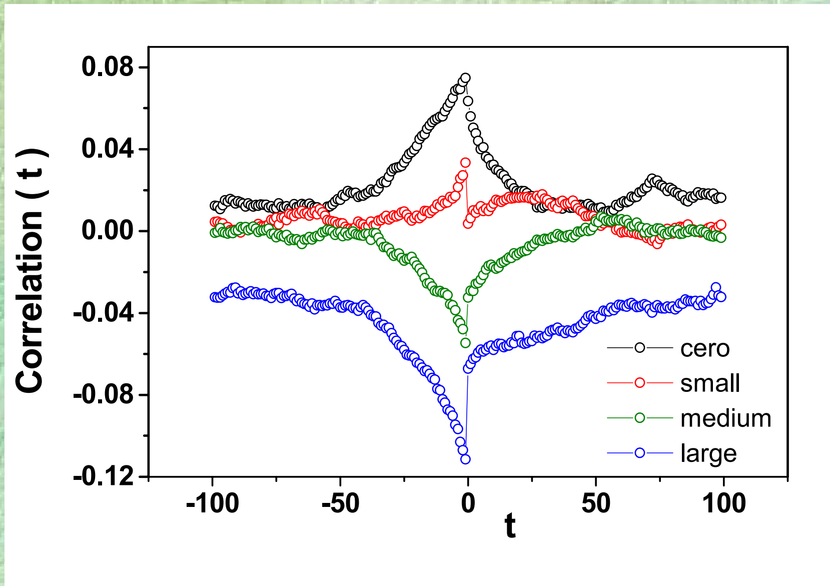


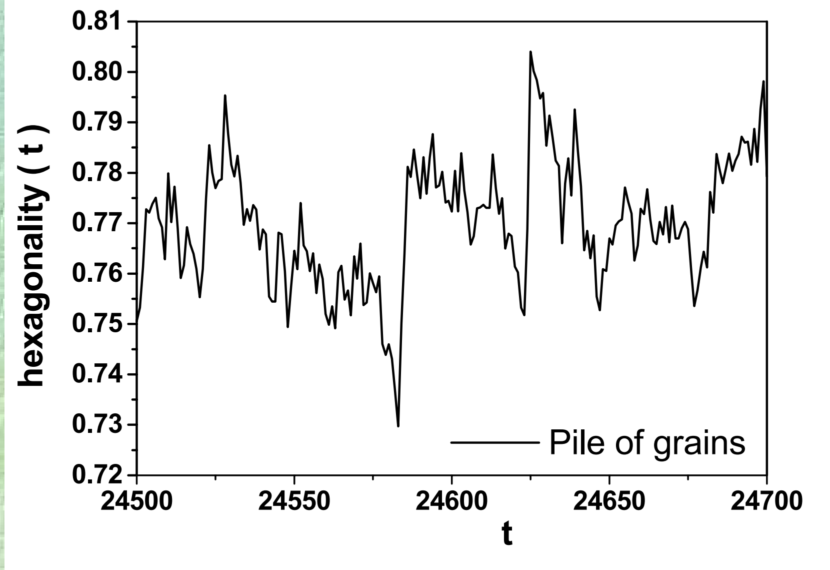


Pile of grains

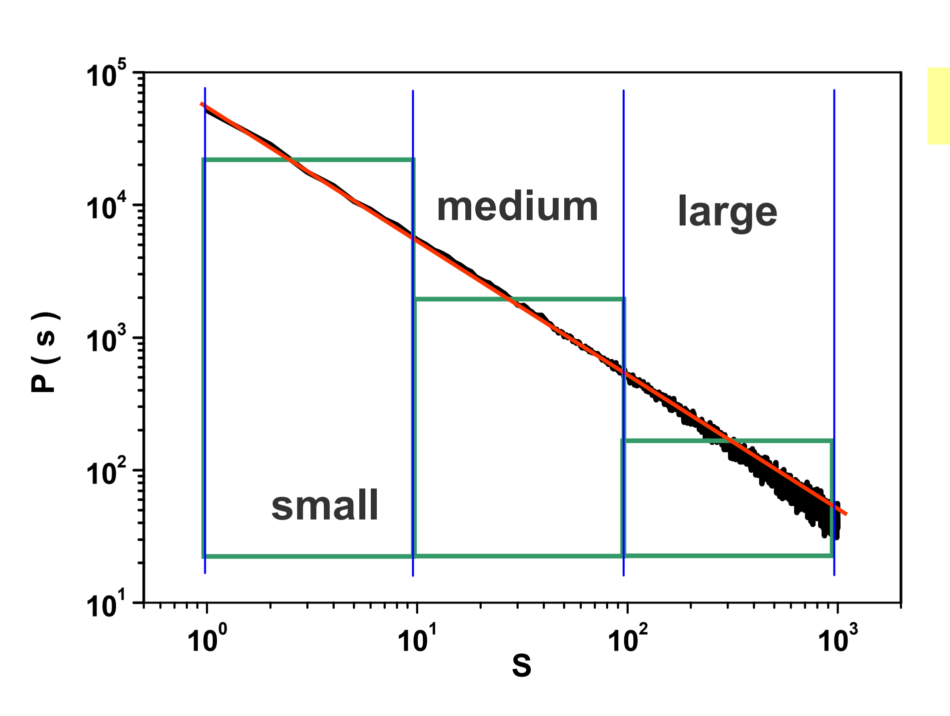
internal avalanches
&
Hexagonality (compactness)







Some probabilities



m = -1

$$area = \int_a^b P(s)ds = \int_a^b s^{-1} ds$$
$$= \ln(\frac{b}{a})$$

if:

$a = 10^{n-1}$

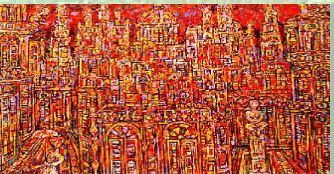
$b = 10^{n}$

$$area = \ln\left(\frac{10^n}{10^{n-1}}\right) = \ln(10)$$ **= const !!**

Prob(L) = Prob(M) = Prob(S) = 1/3

if m = -2

$$area = \int_a^b s^{-2} ds = -\left(\frac{1}{b} - \frac{1}{a}\right) = \left(\frac{1}{10^{n-1}} - \frac{1}{10^n}\right) = \frac{9}{10^n}$$ **~ 10^{-n}**



Ley de Gutemberg - Richter: bM ~ Log(N)

where

M ~Log(Energy) magnitude

damage

b ~ -1

N = number of earthquakes with magnitude ≥ M.

Then

Log [N (E)] ~ - Log E

N (E) ~ E $^{-1}$

P(x) : number of avalanches with magnitude = M

N(x) : number of avalanches with magnitude ≥ M

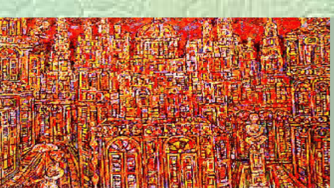
$$P(x) = N(x) - N(x + \Delta x)$$

$$\frac{P(x)}{\Delta x} = -\left[\frac{N(x+\Delta x) - N(x)}{\Delta x}\right]$$

$$P(x) \sim -N'(x)$$

if N (x) = $\mathbf{x}^{-1}$

$$P(x) \sim -\left(x^{-1}\right)' = -\left(-x^{-2}\right) = x^{-2}$$



CONCLUSIONS

1 - Those we already saw related to *off the edge* avalanches.

2 - Although the avalanche prediction is impossible (in a SOC scenario) considered only the temporal sequence of avalanches; seems to be possible to find signs in the structure of the system that precede an avalanche happening.