Avalanche Prediction in a Self-Organized Pile of Beads

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(Received 28 July 2008; published 19 February 2009)

It is a common belief that power-law distributed avalanches are inherently unpredictable. This idea affects phenomena as diverse as evolution, earthquakes, superconducting vortices, stock markets, etc., from atomic to social scales. It mainly comes from the concept of "self-organized criticality" (SOC), where criticality is interpreted in the way that, at any moment, any small avalanche can eventually cascade into a large event. Nevertheless, this work demonstrates experimentally the possibility of avalanche prediction in the classical paradigm of SOC: a pile of grains. By knowing the position of every grain in a two-dimensional pile, avalanches of moving grains follow a distinct power-law distribution. Large avalanches, although uncorrelated, are on average preceded by continuous, detectable variations in the internal structure of the pile that are monitored in order to achieve prediction.

DOI: 10.1103/PhysRevLett.102.078701 PACS numbers: 05.65.+b, 45.70.Ht, 89.75.−k

In the last two decades much effort has been devoted to understanding the ubiquity of scale invariance in nature. The best known attempt so far, although controversial, has been self-organized criticality (SOC) [1,2] in which the competition between a driving force that very slowly injects energy and a dynamics of local thresholds, can drive the system into a critical state where a minor perturbation can trigger a response (avalanche) of any size and duration. A major goal of these kinds of models is to gain understanding of processes ruled by scale invariance that eventually have catastrophic events (evolution [3], earthquakes [4,5], stock markets [6], solar flares [7], superconducting vortices [8,9], etc), in order to predict them. Since unpredictability is an essential feature of critical processes evolving through power-law distributed avalanches, the possibility of prediction seems unlikely. However, prediction is —in principle— possible, because the system is not at, but just close to, the critical state [10,11]. Some cellular automaton models of earthquakes have analyzed the predictability of very large avalanches (responsible for the cutoff on a power-law distribution) [12] and also precursors of large events have been reported in dissipative or hierarchical lattices [13,14]. However, no experimental results have been presented so far concerning the prediction of avalanches inside a regime of power-law distributed events, in slowly driven systems, and still many researchers think that if earthquakes correspond to a SOC process, they are inherently unpredictable [15,16].

We present an experiment where avalanches display a power-law distribution with an exponent equal to -1.6 and where precursors to large events have been found. We use a setup similar to that reported by Altshuler *et al.* [17] where SOC behavior was observed. However, the present experiment is the first one having single grain resolution in the measurement of the avalanches that take place not only at the free surface [18,19] or falling off the system [17,20–22], but also in the bulk of the pile. A base, consisting of a 60 cm long row of 4 ± 0.005 mm steel spheres separated from each other by random (0, 1, 2 or 3 mm) spacing, is glued to an acrylic surface and sandwiched between two parallel vertical glass plates 4.5 mm apart. The same steel beads are delivered one by one from a height of 28 cm above the base and at its center, resulting in the formation of a quasi-two-dimensional pile. The extremes of the base are open, leaving the beads free to abandon the pile. After a bead is delivered, the system waits for a few seconds in

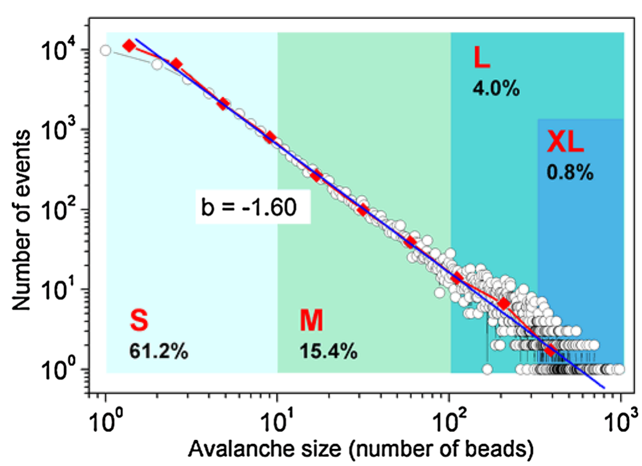

FIG. 1 (color online). Distribution of avalanche size (open circles). These points have been averaged with a logarithmic binning (diamonds). Avalanches are classified as small (S), medium (M), large (L), and very large (XL) considering logarithmic bins. The percentage of every type of avalanche related to the total number of dropping events is also displayed.

order to guarantee that all the relaxation effects in the pile are finished. The pile is then recorded with a Canon D20 digital camera at a resolution of 21 pixels/bead diameter, followed by the dropping of a new bead. One experiment contains more than 55 000 dropping events with a total duration of more than 310 hours. The first 4500 events before the pile reaches a stationary state are not included in the statistics. The average number of beads in the pile is 3315. By processing the images, the centers of all the beads are found and the avalanche size is defined as the number of beads moving between two consecutive dropping events (we include here the beads that fall off the pile). We have assumed that one bead has moved when the coordinates of its center, "projected" on the consecutive image, do not indicate any neighboring center at a distance less than or equal to $1/7$ of the bead diameter. The distribution of avalanche sizes is displayed in Fig. 1. Avalanches span over three decades in a log-log plot and follow a power law in almost all of this range. The graph can be divided (without losing generality) in three equally spaced zones. So, in considering a distribution that spans over $3n$ decades, avalanches smaller than n are considered small, those lying between n and $2n$ are medium, and those greater than $2n$ are large. We focus our attention on the large ones (size $>$ 100) and we look for precursors in the structure of the system. The very large avalanches (size $>$ 316), which have a very small probability of occurrence, are also analyzed.

The temporal autocorrelation of large avalanches reads as

$$C_A(t) = \frac{\sum[s(\tau)s(\tau+t)] - \langle s(\tau)\rangle^2}{\sum[s(\tau) - \langle s(\tau)\rangle]^2}, \tag{1}$$

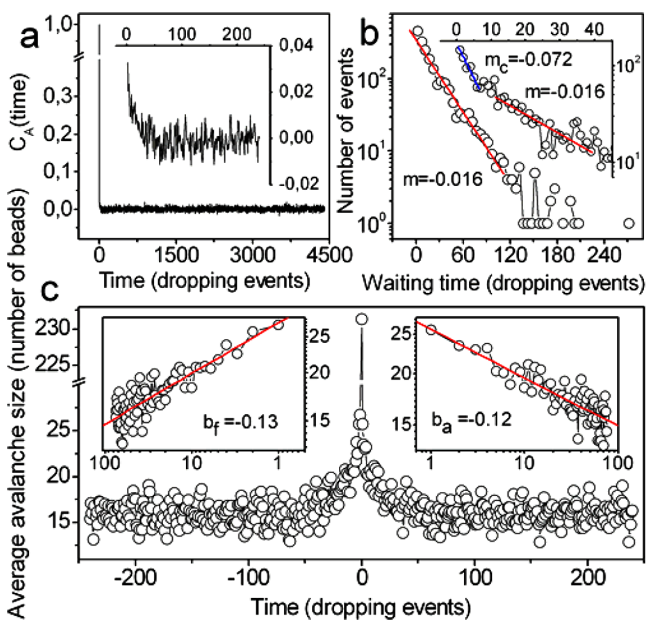

FIG. 2 (color online). Analysis of the avalanche time series. (a) Autocorrelation of the large avalanche time series. (b) Distributions of waiting time between large avalanches. (c) Average avalanche size around a large avalanche.

where $s(\tau)$ equals unity if the avalanche is large, and zero in all other cases. The unit of time corresponds to one dropping event, also called a step. The uncorrelated character of large (L) avalanches is displayed in Fig. 2(a). The waiting time between them follows an exponential distribution [Fig. 2(b)]. This implies the presence of a characteristic waiting time, equal to 26 ± 2 steps, indicating the average time between large events. Very large avalanches (XL) behave in a similar way, with a characteristic waiting time equal to 132 ± 9 steps. Concerning predictability, it is interesting to note that the earthquake scenario is better than the one presented here, in the way that the waiting times between events follow a power-law distribution, allowing some global, or long term, forecast [23,24]. Up to this point in the analysis, we have a self-organized system with uncorrelated avalanches whose sizes follow a power-law distribution. So it seems impossible to predict the moment when a large avalanche is going to happen. However, a slight decay larger than the background noise can be noticed for the first 20 steps in the inset of Fig. 2(a).

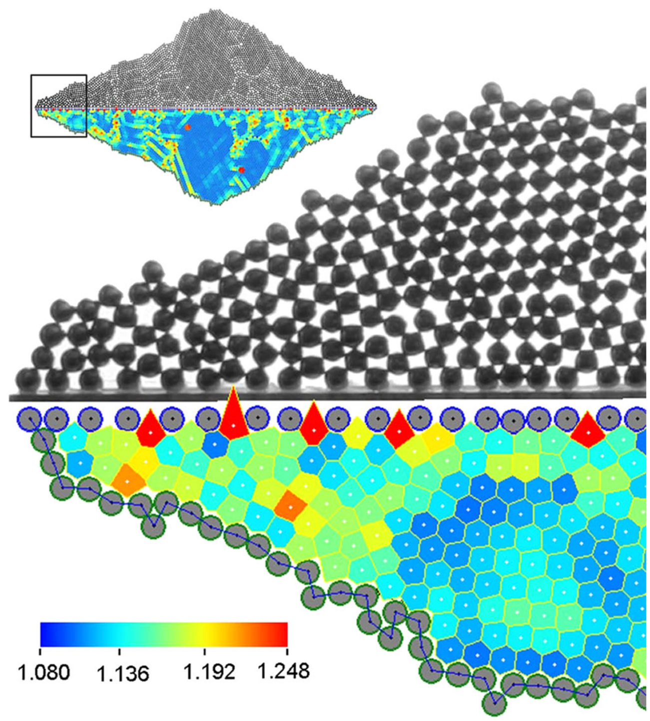

FIG. 3 (color online). Internal structure of a pile. Close-up of a portion of the pile with a reflection (below) in the space of the shape factor ζ. After finding the center of the beads and making a voronoi triangulation for the internal ones (i.e., excluding those in the base and those joined with a line at the pile's surface) we define $\zeta = C^2/4\pi S$, where C is the perimeter and S the area of each voronoi cell. The shape factor ζ is a measure of the local disorder in the pile [for example for a regular hexagon $\zeta = 1.103$ (highest order in the pile) and for a square $\zeta = 1.273$]. Inset: The whole pile, where a rectangle indicates the close-up area. Notice the small values of ζ at the center of the pile, indicating hexagonal packing which results from the "thermal" effect of beads being added from above.

This is an indication of a time clustering of avalanches, and a steeper slope m_c for the small values of the waiting time in the inset of Fig. 2(b) is also a sign of it. This temporal clustering is analyzed in Fig. 2(c): around a large event there is an average increment of the avalanche size. The best fit of it corresponds to power laws (insets) for both ''foreshocks'' and ''aftershocks''. The exponents of the power laws are very low (-0.13 and -0.12), making it impossible to predict main shocks by analyzing possible foreshocks. Any comparison with the Omori law [25] for earthquakes can be done just qualitatively due to the fact that in our experiment there is only one avalanche per unit time.

To predict, in the short-term, when a large avalanche is going to happen, we analyze the corresponding changes in the internal structure of the pile. As the position of the centers of all the particles at every step of the experiment is known, we are able to define several structural variables and analyze how they evolve during time, particularly in the neighborhood of a large avalanche. In this Letter we focus our study on just two: the size of the system, defined as the number of beads in the pile, and the shape factor ζ [26], which is a measure of the local disorder in the system (Fig. 3). It is expected that during a big avalanche, some of the grains will eventually abandon the pile, and therefore the size of the system should, on average, decrease. If so, before a large event, the pile, and thus the amount of energy ready to be released during the avalanche has to be large.

The temporal correlation function

$$C(t) = \frac{\sum[s(\tau)x(\tau + t)] - \langle s(\tau)\rangle\langle x(\tau)\rangle}{\sqrt{\sum[s(\tau) - \langle s(\tau)\rangle]^2 \sum[x(\tau) - \langle x(\tau)\rangle]^2}}, \quad (2)$$

where $x(\tau)$ corresponds either to the temporal series of the size of the pile [Fig. 4(a)] or to the average shape factor [Fig. 4(b)], displays the behavior of these structural variables in the vicinity of a large avalanche. The number of beads in the pile [Fig. 4(a)] behaves as ''expected'' and approximately 50 dropping events (steps) before a large avalanche the size of the pile, on average, suffers a slight increment and then a continuous decrement (foreshocks zone) until the avalanche takes place. During the avalanche the pile's size jumps down. It then continues decreasing for around 25 steps (aftershocks zone), but starts to grow again soon after due to the addition of new grains from the top. The continuous variation displayed by the disorder of the pile before a large avalanche is much clearer [Fig. 4(b)] and approximately 50 steps before a large event, the average disorder continuously increases until the avalanche takes place. Then the pile reorganizes itself, but it gets trapped in an intermediate level of disorder. In the aftershocks zone the disorder increases, and after that, the pile slowly evolves into more organized states. Even locally it can be noticed as a very good match between the zones involved in large avalanches, and the increase of disorder before the avalanche takes place [Figs. 4(c) and 4(d)]. The

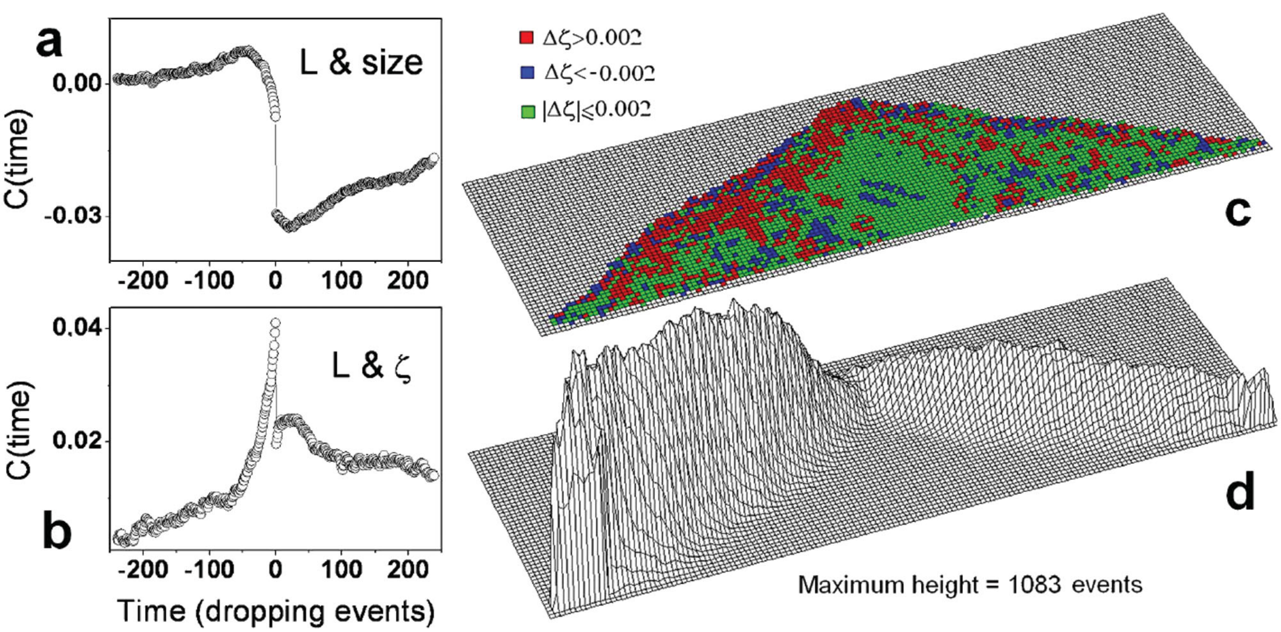

FIG. 4 (color). Correlation between avalanches and internal structure. (a) Temporal correlation between the large avalanches and the size of the system. (b) Temporal correlation between the large avalanches and the average shape factor ζ. (c) Difference between the local averages of the ζ values at one step before a large avalanche and at 50 steps before a large avalanche. Red indicates that the disorder increases, blue an increase in the order, and green displays no variations in ζ. A cursory inspection shows that the red color predominates over the blue . The cumulative number of sites involved in large avalanches is displayed at (d). The match between the red color in (c), and the landscape in (d), corroborates that on average, the pile suffers an increment of the disorder before a large avalanche takes place.

observed asymmetry is a consequence of a slight tilt of the ordered (hexagonal) block at the center of this particular pile, favoring the movement of grains to the left of the system. The same general behavior of structural changes in the pile preceding large events has been observed in other piles developed under similar conditions.

In order to try an actual prediction, we define an alarm for an upcoming large (L) event. The alarm is in its *on* state when the differences of the spatial average of the shape factor between the current time and 50 steps before the current time is larger than zero; i.e., if $\zeta(t) > \zeta(t-50)$ then we predict an avalanche of size L at the time $t+1$. After 50 000 events, the alarm is *on* $51 \pm 3\%$ of the total time, and $62 \pm 4\%$ of the large avalanches take place during this situation. The same analysis for the very large (XL) avalanches gives $64 \pm 7\%$ of the events happening under an alarm that is *on* $51 \pm 3\%$ of the total time. The errors have been calculated by dividing the data in 10 different portions of 5000 events each. The mean value and the standard deviation are the values reported before. Perhaps this 62% is not a very impressive percent (in a random process 50% of the events will occur under an alarm that is *on* 50% of the time), but the first criterion we have used for turning the alarm *on* is very simple. If we use a bit more sophisticated criterion, better results are obtained. For example, taking into account the existence of aftershocks, we can define a second alarm that is turned *on* immediately after a large (L) event and has a duration of 12 steps. If the system is under an alarm (either this second alarm or the original one) $50.0 \pm 0.1\%$ of the total time (in order to get this value the thresholds of differences of shape factors in the original alarm have to be readjusted), $65 \pm 4\%$ of large avalanches take place when the alarm is *on*. The results given by our naïve criterion for predicting avalanches, based only on Fig. 4(b), are somehow poor. However, they are able to demonstrate that by means of the analysis of the correlation between large avalanches and global structural variables in the system, it is possible to achieve some predictability. We also believe that this percentage of success can be increased by adding more information [for example, the size of the pile: Fig. 4(a)], and by using smarter techniques such as pattern recognition tools.

We have presented a self-organized pile of beads that evolves through uncorrelated avalanches distributed following a power law, and where the analysis of the time series of events does not give enough information for developing any forecast of large events by itself. However, on average, some global structural variables display continuous, significant and detectable variations preceding large events. A similar behavior has been noticed in other numerical models of earthquakes and superconducting vortices [27]. This indicates that, in these self-organized situations, large avalanches require the building-up of certain conditions in the structure of the system. By monitoring this developing process these large events can, in principle, be forecasted.

We thank O. Duran, G. Løvoll, D. Molenaar, N. Arnesen, S. Santucci, L. Vanel, and K. P. Zabielski. This work was supported by NFR, the Norwegian Research Council, through a grant by Petromax and SUP.

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